FernUniversität in Hagen

ENTROPY-DRIVEN PORTFOLIO SELECTION

a downside and upside risk framework

Wilhelm Rödder, Ivan Ricardo Gartner, Sandra Rudolph

Diskussionsbeitrag Nr. 437

April 2009

Diskussionsbeiträge der Fakultät für Wirtschaftswissenschaft

der FernUniversität in Hagen

Herausgegeben vom Dekan der Fakultät

ENTROPY-DRIVEN PORTFOLIO SELECTION
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Wilhelm Rödder
Department of Operations Research, University of Hagen, 58084 Hagen, Wilhelm.Roedder@FernUni-Hagen.de

Ivan Ricardo Gartner
Department of Operations Research, University of Hagen, 58084 Hagen, Ivan.Gartner@FernUni-Hagen.de

Sandra Rudolph
Department of Operations Research, University of Hagen, 58084 Hagen, Sandra.Rudolph@FernUni-Hagen.de

In modern portfolio theory like that of Markowitz or Sharpe the investor follows a mean/variance-rationality. Even the founders of this theory observed unsatisfactory results because of symmetrical risk measures like variance or standard deviation. Post-modern theory then considers downside risk measures and takes into consideration the investor's specific goals. In this contribution we follow these ideas, but use an information theoretical inference mechanism under Maximum Entropy and Minimum Relative Entropy, respectively. The approach results in a high performance Expert System under the shell SPIRIT, combining an index model with the new method. For three DAX listed blue chips and for varying risk attitudes of the investor the system's portfolio selection capacity is compared to that of classical Markowitz & Sharpe optimization.

Key words: Finance, Artificial intelligence, Expert systems, Portfolio selection

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1. Introduction

In modern portfolio selection theory founded by Markowitz (1952), risk is understood as the total variability of returns. Our intuition tells us this concept to be erroneous as it equally suggests penalties for up and for down deviations from the mean. Already a few years later Sharpe (1964) criticizes the risk concept and also Markowitz (1959) recognizes its limitations. The latter faces these difficulties by recommending a downside risk measure, the so called semi-variance. This basic idea has been widely accepted, see Bawa (1982) for a good bibliography.

Thus the idea of avoiding negative and seeking positive risks is not new at all. Any investor follows these principles, even though with different risk attitudes, of course. He or she might be aggressive, moderate or conservative, cf. Elton & Gruber (1991). In classical theories like that of Markowitz this attitude is modelled as so called risk aversion or risk acceptance parameters, but in a somewhat plain manner.

In this contribution we build a model in which

- the expected security returns for a holding period are predicted by means of a simple linear Index Model, being the DAX the respective regressor.
- for any index prediction the respective return distributions for the securities are informed to the Expert System Shell SPIRIT (2008). SPIRIT by means of this information provides a good portfolio without any calculations of classical risk measures like variance or standard deviation. It simply and solely suggests an unbiased portfolio applying a Maximum Entropy/Minimum Relative Entropy (MaxEnt/MinREnt) approach upon upside and downside risks for all securities.
- the risk attitude of any investor can be specified explicitly.

The main purpose of this paper is the demonstration of the method's power. This is done by simulating a 1½-years period from July 2006 until December 2007, monthly selecting portfolios of the three stocks BASF (BASF.F), Volkswagen (VOW.F), MAN (MAN.F) and closing the position after a one month holding; here F denotes Frankfurt stock exchange. The accumulated returns over this period are compared to those of classical Markowitz & Sharpe approaches.

Section 2 is headed "Mathematical and Logical Prerequisites". In Subsection 2.1 we sketch the well-known Index Model, in 2.2 we list the Markowitz and Sharpe models employed in the remainder of this contribution. 2.3 then is dedicated to entropy-driven knowledge acquisition and information processing. For that we introduce the syntax and semantics of probabilistic conditionals (2.3.1), develop the principle of MaxEnt/MinREnt-reasoning as supported by the Expert System Shell SPIRIT (2.3.2), and finally show elements of the portfolio selection process (2.3.3). This subsection is essential since it makes the reader familiar with the entropy approach. Section 3 then compares mean/variance (MV)-based portfolio management to that of the new method. In 3.1 we relate on historical data and the test design, in 3.2 we present numerical results for either type of selection mechanism, and 3.3 shows the simulated portfolio returns for the 1½-years control period. Section 4 is a resume and focuses on further research.

2. Mathematical and Logical Prerequisites

2.1 The Index Model

Index Models serve as a means to predict securities' future behaviour from historical data, cf. Sharpe (1963). In their simplest form stocks are linearly regressed to some index like DOW JONES, EURO STOXX, DAX or others.

$$R_{stock} = \alpha_{stock} + \beta_{stock} \cdot R_{index} + \varepsilon_{stock} \quad (1)$$

is the well-known equation, being R_{stock} and R_{index} random periodical returns, α and β the respective regression coefficients and ε_{stock} the stock's residual, the latter with mean 0 and standard deviation σ. Once $\hat{\alpha}$ $\hat{\beta}$, $\hat{\sigma}$ are estimated from historical data, any periodical index prediction r_{index} permits the determination of the corresponding distribution of $R_{stock}(r_{index})$ with conditioned mean $\hat{r}_{stock} = \hat{\alpha} + \hat{\beta}_{stock} \cdot r_{index}$ and standard deviation $\hat{\sigma}$ – homoskedastic model. In the remainder of this contribution equation (1) will serve as a means for calculating conditioned distributions and expectations. More on that in Section 3.

2.2 Classical MV-Models

In this subsection we sketch the classical MV-Models of Markowitz (1987) and Sharpe (1964) and show their respective relations.

Let R, R_j, j = 1,…,J be random period returns of a portfolio and of its respective risky securities.

Let r, r_j, j = 1,…,J be the returns' means and σ^2, **V** their variance and covariance-matrix.

Let r_0 be the return of a risk-free security and λ a risk aversion parameter.

If now w_j, j = 1, …,J are the stocks' weights in the portfolio and $\boldsymbol{w}$ their corresponding vector then $\sigma^2 = \boldsymbol{w}^T \mathbf{V} \boldsymbol{w}$ is the portfolio's variance and σ its standard deviation. Classical portfolio selection considers the following optimization problems (2) to (5).

$$\max Y = \sum_j w_j \cdot r_j (- r_0) - \lambda \cdot \sigma \qquad \text{s.t.} \quad \sum_j w_j = 1, w_j \geq 0 \quad (2)$$

is a parametric optimization problem which for $\lambda \geq 0$ yields all MV-efficient portfolios, 1 stands for a unit investment capital. The constant r_0 in (1) serves as a mere link to equation (3), as will be clarified in the next paragraph.

$$\max\ \Theta = \frac{\sum_{j} w_j \cdot r_j - r_0}{\sigma} \qquad \text{s.t.} \quad \sum_{j} w_j = 1,\ w_j \geq 0 \tag{3}$$

maximizes the expected excess over the risk-free security return per risk unit. For Θ^*= max Θ the respective portfolio is called a tangent portfolio, for its special properties cf. Albrecht & Maurer (2005), pp. 278 - 282. (2) and (3) are strongly related: If Θ^*, $\boldsymbol{w}^*$ is optimal for (3) then for $\lambda = \Theta^*$ the weights $\boldsymbol{w}^*$ also are optimal for (2). We shall make use of this fact in Section 3, the obvious proof is left to the reader. Note that for negative expectations, (3) minimizes losses per risk unit.

In (2) and (3) the portfolios only consist of risky securities, in the next two equations a risk-free investment with weight w_0 is explicitly permitted:

$$\max Z = w_0 \cdot r_0 + \sum_{1}^{n} w_j \cdot r_j (- r_0) - \lambda \cdot \sigma \qquad \text{s.t.} \quad w_0 + \sum_{j} w_j = 1;\ w_0, w_j \geq 0. \tag{4}$$

By assumption the risk-free security's return r_0 is deterministic and hence uncorrelated to any risky one. So the variance of the portfolio $w_0 \cdot r_0 + \sum_{1}^{n} w_i \cdot R_i$ is σ^2 and its standard deviation σ, cf. Albrecht & Maurer (2005), p. 203. Note that for a great λ maximizing (4) means minimizing σ. This leads to a great w_0, the investor prefers the risk-free alternative.

$$\max\ \Delta = \frac{w_0 \cdot r_0 + \sum_{j} w_j \cdot r_j - r_0}{\sigma} \qquad \text{s.t.} \quad w_0 + \sum_{j}^{n} w_j = 1;\ w_0, w_i \geq 0 \tag{5}$$

maximizes the Sharpe ratio of portfolios including the risk-free security. (4) and (5) again are strongly related, as are (2) and (3): If Δ^*, w_0^*, $\boldsymbol{w}^*$ is optimal for (5) then for $\lambda = \Delta^*$ the weights w_0^*, $\boldsymbol{w}^*$ also solve (4). We shall make use of this fact in Section 3, the straightforward proof again is left to the reader.

In Section 3 we shall determine optimal weights for either model, (2) until (4), and for varying λ's in the parametric problems.

2.3 Entropy-based Portfolio Selection

2.3.1 Syntax and Semantics of Probabilistic Conditionals

B|A is the conditioned proposition "B given A". Such conditionals are a constituent of human thinking. The more often in our memory Bs are true given that As are true, the more we know and the better we can conclude facts from other facts. Modern Expert Systems like HUGIN 6.9, www.hugin.com (2008), Netica 4.02, www.norsys.com (2008), SPIRIT 3.7, www.xspirit.de (2008), make use of such conditional dependencies of facts from other facts and even allow a complex network of such dependencies to infer conclusions from evidently true premises.

The statements

- BUY = *no*, given that the carmaker VOW very likely will have a *bad performance*
- BOND = *yes*, given that the INVESTOR is *conservative*

are such conditionals. Conditionals help to model highly complex economical and technical contexts, and even support the decision maker in likewise complex decision situations, see Kulmann (2002), Schramm & Fronhöfer (2001), Rödder et al. (2006), Reucher & Kulmann (2007). So when a system comes to know that the carmaker VOW evidently will have a bad performance it will recommend not to buy the share, once it has internalized the above rule. Similarly it will conclude to buy bonds if the investor is certainly conservative. How this can be done in large scale applications is the subject of this and the following subsections. First of all, however, we must define the syntax and semantics of conditionals.

Let $\mathcal{V} = \{V_1, \ldots, V_M\}$ be a finite set of finite valued variables with attributes v_m of V_m. We often use mnemic upper case names for the variables and lower case names for the attributes. BUY = *no* or INVESTOR = *conservative* are typical examples of variables and their respective attributes. Formulas of the type $V_j = v_j$ are literals. They are atomic propositions, which can be true (t) or false (f) under a certain interpretation. From such literals, elements of a propositional language L are formed by the junctors $\wedge$ (and), $\vee$ (or), $^{-}$ (not) and by parentheses; such elements are denoted by A, B, C, … . Complete conjuncts of literals we write as $v = v_1 \ldots v_m$, for short. | is the binary conditional operator. The formula B|A is a conditional and hence an element of the conditional language L|L. BUY = *no* | INVESTOR = *conservative* is such a conditional. B|A is equal to B for a tautological A, such unconditioned formulas are called facts. For further details on the syntax cf. Rödder et al. (2006), e.g.

A probabilistic conditional or rule is of the form B|A [x], here x expresses the probability of B|A to be true in a certain context. If now we have

$\mathcal{R} = \{B_i|A_i\ [x_i],\ i = 1,\ldots,I\},$ (6)

a set of probabilistic conditionals valid in a certain real world context, it is the invitation to find an appropriate probability measure P on $\mathcal{V}$, i.e. a measure respecting the conditionals' probabilities. Such a P is considered an epistemic state on $\mathcal{V}$ representing a context and is called a semantic model of (6).

2.3.2 MaxEnt/MinREnt - Principle and the Expert System Shell SPIRIT

The epistemic state P of the last subsection is not uniquely determined by the conditionals or rules in $\mathcal{R}$, in general. It needs additional postulations to establish a unique P from such rules. These postulations might be a structural netting of rules together with certain independence assumptions like in Bayes-nets, Jensen (1996), or other principles. The most famous of such principles is the MinREnt-principle.

For two measures P and Q, $R(Q,P) = \sum_{v} Q(v) \cdot ld(Q(v)/P(v))$ is called Relative Entropy of Q with respect to P. R measures the overall change of conditional probabilistic structure from P to Q, Shore & Johnson (1976), Paris & Vencovská (1990), Kern-Isberner (1997), Rödder (2000). R(Q,P) is equal to the well-known Kulback-Leibler divergence K(Q||P), see Csiszàr (1975).

If now $\mathcal{R} = \{B_i|A_i[x_i],\ i = 1,\ldots,I\}$ like in equation (6), represents conditional knowledge about a real world situation then solving equation (7) is knowledge acquisition under the MinREnt-principle:

$P^* = \arg\min R\ (Q,P^0) \quad \text{s.t.} \quad Q(B_i|A_i) = x_i \quad i = 1,\ldots,I.$ (7)

P^* respects all conditional knowledge and avoids not intended dependencies between the involved variables. It does so keeping the unstructured uniform distribution P^0 as far as possible. Minimizing the Relative Entropy like in (7) is equivalent to Maximizing Entropy. The MinREnt-principle coincides with the MaxEnt-principle for this special case, cf. Meyer (1998), p. 69.

P^* represents acquired knowledge under this principle. In Section 3 we shall make the Expert System Shell SPIRIT acquire all disposable knowledge about a portfolio real world situation.

Once the system has internalized all this knowledge, it is ready to support investors with different risk attitudes in selecting good portfolios.

If now certain facts become evident, the knowledge base P^* permits to conclude facts and to recommend decisions. The respective process under MinREnt is called query and response:

$$P^{**} = \arg\min R(Q, P^*) \quad \text{s.t.} \quad Q(F) = 1$$
$$P^{**}(G) = y. \tag{8}$$

The first part adapts the knowledge base P^* to the evident situation F and the second part is a mere evaluation of the query's G probability under this adaptation. (8) is a very special case of much more general virtual query processing which is beyond the scope of this paper. The interested reader is referred to Rödder et al. (2006), however.

Considering the portfolio selection problem, F might be the evident fact that the investor is conservative and the carmaker VOW very likely will have a bad performance. Then the mathematical equivalent to these verbal statements is Q(INVESTOR = *conservative* ∧ VOW = *bad performance*) = 1. Equipped with such situative information the knowledge base P^* – temporarily – is transformed into P^{**} thus permitting the calculation of a response's probability. P^{**}(BUY = *no*) = 0.95 would be such a probability.

The Expert System Shell SPIRIT supports knowledge processing as presented here. The program is written in JAVA and so runs under any operating system. It is able to handle umpteen variables and hundreds of probabilistic rules. For very large knowledge bases it automatically breaks down the global P^* to marginal distributions in so called Local Event Groups. The user might process simple queries like in (8) or complex virtual queries. For the latter the shell offers a good folder management so as to support the user in handling basic and situative knowledge.

Once the user provides inconsistent or not compatible knowledge to the system, the shell offers an inconsistency check and suggests corrections.

Besides learning from rules like in (7) the shell also permits learning from real world data: inductive learning.

SPIRIT can be downloaded from www.xspirit.de, it is widely auto explicative, for more details see Rödder et al. (2006). For models like the one considered in this contribution all calculations are realized in a few milliseconds on any modern PC.

2.3.3 Portfolio Selection by SPIRIT

In this subsection the variables and representative probabilistic conditionals, as a control mechanism for the securities' weights in the portfolio will be presented. Later, in Section 3, all this will be supplemented by real world data.

We start with the definition of the variables and their attributes.

The six variables DAX, SECUR_0, SECUR_1, SECUR_2, SECUR_3 and PORTFOLIO stand for the index DAX, for the risk-free security and for the stocks BASF, VOW, MAN. Each of these variables might assume the same values or attributes:

rmm, *rm*750, *rm*450, *rm*150, *rp*150, *rp*450, *rp*750, *rpp*. These discrete attributes denominate possible return classes from very low to very high: *r*eturn *m*inus *m*inus until *r*eturn *p*lus *p*lus. The classes will be specified further in Section 3.

The six variables RISK_1, RISK_2, RISK_3, PERFORM_1, PERFORM _2, PERFORM _3 represent risk categories. For a security, the risk is *high* if the respective returns fall into {*rmm*, *rm*750}, it is considered *middle* for returns in {*rm*450, *rm*150}, and *low*, otherwise. Similarly PERFORM = *high* corresponds to {*rp*750, *rpp*}, PERFORM = *middle* to {*rp*150, *rp*450}, and PERFORM = *low* to the remaining return classes.

AGE and MARITSTAT indicate characteristics of the investor, the former with attributes *young*, *middle*, *old* and the latter with *single*, *married*, *widowed*. The investor's risk attitude is described by the variable RISKATT and its values are *aggressive*, *moderate* and *conservative*. The variable W with its attributes w_0, w_1, w_2, w_3 controls the securities' weights in the portfolio. And finally the utility variable U allows the attribution of numerical returns to each of the above return classes, as will be demonstrated later on.

There are eight groups of probabilistic facts and conditionals which inform respective knowledge about the portfolio context to the system.

G1: The first group attributes probabilities to DAX-values. DAX = *rmm*[0.054] is such an attribution. From these probabilistic facts the system learns observed historical DAX behaviour.

G2: The second group is a discrete approximation of the linear regression between index and securities. SECUR_1 = *rm*150|DAX = *rm*450 [0.279] is a typical attribution. This is done for all DAX-values and all stocks. For the risk-free security we have SECUR_0 = *rp*150 [1], as the monthly return falls into this class.

G3: The third group is similar to G2, but now portfolio returns are attributed to conditioned DAX returns. PORTFOLIO = *rm*150|(DAX = *rm*450 $\wedge$ W = w_1) [0.279] is a typical attribution. This is done for all DAX-values and all stocks. For the risk-free security we have PORTFOLIO = *rp*150|W = w_0 [1].

Theoretical Excursus: Please notice that the rules in G3 superpose all linear regression approximations like in G2 and form a portfolio assemble, once the w_j's probabilities are established. More precisely:

If rk and rk′, k,k′ = 1,…,8 denote the respective return classes, if u_k denote their monetary utilities and if P^* is a MinREnt optimal probability measure, then because of

$$P^*(\mathrm{PORTFOLIO} = r\mathrm{k})$$
$$= \sum_j \sum_{k'} P^*\left(\mathrm{PORTFOLIO} = r\mathrm{k} \middle| \mathrm{DAX} = r\mathrm{k'} \wedge \mathrm{W} = w_j\right) \cdot P^*\left(\mathrm{DAX} = r\mathrm{k'} \wedge \mathrm{W} = w_j\right)$$

we conclude

$$\sum_k u_k \cdot P^*(\mathrm{PORTFOLIO} = r\mathrm{k})$$
$$= \sum_k u_k \cdot \sum_j \sum_{k'} P^*\left(\mathrm{PORTFOLIO} = r\mathrm{k} \middle| \mathrm{DAX} = r\mathrm{k'} \wedge \mathrm{W} = w_j\right) \cdot P^*\left(\mathrm{DAX} = r\mathrm{k'} \wedge \mathrm{W} = w_j\right)$$
$$= \sum_j \sum_{k'} P^*\left(\mathrm{DAX} = r\mathrm{k'} \wedge \mathrm{W} = w_j\right) \cdot \sum_k u_k \cdot P^*\left(\mathrm{PORTFOLIO} = r\mathrm{k} \middle| \mathrm{DAX} = r\mathrm{k'} \wedge \mathrm{W} = w_j\right)$$
$$= \sum_j \sum_{k'} P^*\left(\mathrm{DAX} = r\mathrm{k'} \wedge \mathrm{W} = w_j\right) \cdot \sum_k u_k \cdot P^*\left(\mathrm{SECUR\_j} = r\mathrm{k} \middle| \mathrm{DAX} = r\mathrm{k'}\right)$$
$$= \sum_j \sum_{k'} P^*\left(\mathrm{W} = w_j \middle| \mathrm{DAX} = r\mathrm{k'}\right) \cdot P^*(\mathrm{DAX} = r\mathrm{k'}) \cdot \sum_k u_k \cdot P^*\left(\mathrm{SECUR\_j} = r\mathrm{k} \middle| \mathrm{DAX} = r\mathrm{k'}\right)$$

Now evidentiating $Q(\mathrm{DAX} = r\mathrm{k'}) = 1$ for a fix $\mathrm{k'}$ like in (8) yields some $P^{**}(\mathrm{k'})$. Suppressing the index $\mathrm{k'}$ we get

$$\sum_k u_k \cdot P^{**}(\mathrm{PORTFOLIO} = r\mathrm{k}) \tag{9}$$
$$= \sum_j P^{**}(\mathrm{w}_j) \cdot \sum_k u_k \cdot P^{**}(\mathrm{SECUR\_j} = r\mathrm{k})$$

Equation (9) shows the expected portfolio return – left side – to be equal to the weighted sum of expected security returns – right side. It remains to determine P^{**} in such a way that it provides good weights P^{**} (w_j). That is what G4 until G7 are about.

G4: The fourth group attributes monetary utilities in percent of the unit capital to all return classes. U = -4.5| PORTFOLIO = *rm*450 [1] is a typical attribution. These values are the class means and represent respective average returns.

G5: The fifth group links investor characteristics with risk attitudes. RISKATT = *aggressive*| (AGE = *young* ∧ MARITSTAT = *single*) [1] is a typical rule. These rules are of minor importance in our system and could be modified anytime. Any of these rules should and could be justified empirically.

G6: The sixth group combines the security return classes to risk and performance categories.

(SECUR_1 = *rmm* $\vee$ SECUR_1 = *rm*750)|RISK_1 = *high* [1] or

(SECUR_1 = *rpp* $\vee$ SECUR_1 = *rp*750)|PERFORM_1 = *high* [1]

are typical rules of that kind. If a stock's risk is high, it certainly falls into one of the return classes *rmm* or *rm*750. If its performance is high it certainly represents respective high returns.

G7: These conditionals form the central control mechanism of the security weights as a function of the securities performances and risks.

PERFORM_1 = $\overline{low}$|W = w_1 [1] or RISK_1 = $\overline{low}$|W = w_0 [1]

are typical rules of that kind. Here $\overline{}$ stands for negation, cf. Subsection 2.3.1, and so the first conditional reads: If we buy BASF its performance certainly is not low. The reader is invited to interpret the second rule analogously.

G8: These rules inform the system about the influence of the investor's risk attitude over his/her readiness to buy.

RISKATT = *conservative*|W = $\overline{w_0}$ [0.01]

is a typical rule of this group. It indicates that an investor who does not apply in risk-free bonds, very unlikely is conservative, e. g.

This concludes the presentation of typical conditionals. The portfolio knowledge base amounts to a total of 432 facts and rules.

Theoretical Excursus: The reader must keep in mind that the rules in G6 control the performance and the risk variables, and that the ones in G7 control the portfolio weights (!). So it might be displeasing to find these variables in the conditio rather than in the conclusio of respective rules. The control mechanism is a pretty complicated matter, and so at least for G7 we sketch how it works in a MinREnt framework.

Any rule of the form $\overline{B} \mid A\,[1]$ has the desired effect that for all this rule respecting distributions Q, a decreasing probability Q($\overline{B}$) implies in a decreasing probability of A, too. This is true due to the inequality $Q(\overline{B}) \geq Q(\overline{B}A) = Q(A)\cdot 1$. The conditional PERFORM_1 = $\overline{low}$|W = w_1 [1] recommends not to buy BASF-shares for an evidently unlikely high or middle performance. This is classical probability theory. MinREnt inference is more. Here the rule $\overline{B} \mid A\,[1]$ for an increasing probability of $\overline{B}$ implies in an increasing probability of A, too. Under P^* we have $P^*(\overline{B} \mid A) \cdot P^*(A) = P^*(A \mid \overline{B}) \cdot P^*(\overline{B})$ and hence

$1 \cdot P^{*}(A) = P^{*}(A \mid \overline{B}) \cdot P^{*}(\overline{B})$. If now things like DAX-values and investor characteristics become evident like in (8) in subsection 2.3.2, and if this evidence results in an increasing probability of $Q(\overline{B})$ then the MinREnt adaptation is $1 \cdot P^{**}(A) = P^{*}(A \mid \overline{B}) \cdot P^{*}(\overline{B}) \cdot (Q(\overline{B}) / P^{*}(\overline{B}))$. This kind of evidence adaptation is related on in Rödder (2000), Appendix A, e.g. Under P^{*} a very likely not low performance will always improve the recommendation to buy.

The present subsection related on the conditionals to feed the system with knowledge about the portfolio context. The following section is dedicated to historical securities data, to the construction of a test design for an 18 months comparison of different selection methods, and to the presentation of numerical results.

3. MV - versus MinREnt-Optimization – A Comparison

3.1 Data and Test Design

The data are DAX, BASF, VOW and MAN prices P_t at the beginning of each month t, for January 2002 until June 2006. For each of these months we calculate the respective returns by means of the formula $r_t = \ln(P_t / P_{t-1})$ and finally regress all single securities with the index DAX. With high statistical significance we get the following linear regression equations, being the residuals Gaussian:

$$\begin{aligned} R_{BASF} &= 0.006 + 0.792 \cdot R_{DAX} + \varepsilon_{BASF} && \text{with } \sigma_{BASF} = 0.0392 \\ R_{VOW} &= -0.0001 + 0.809 \cdot R_{DAX} + \varepsilon_{VOW} && \text{with } \sigma_{VOW} = 0.0760 \\ R_{MAN} &= 0.014 + 1.101 \cdot R_{DAX} + \varepsilon_{MAN} && \text{with } \sigma_{MAN} = 0.0812. \end{aligned} \tag{10}$$

The covariance matrix of the three securities for the same time interval amounts to

$$\mathbf{V} = \begin{pmatrix} 0.004476 & 0.003321 & 0.004665 \\ 0.003321 & 0.008694 & 0.005783 \\ 0.004665 & 0.005783 & 0.012153 \end{pmatrix}. \tag{11}$$

The control interval then consists of the months July 2006 until December 2007. For this interval all MV-methods as presented in Subsection 2.2, are applied in the following manner.

- We make a short-term analysis: For each month the DAX-behaviour is predicted by the naïve forecast, taking the last month’s result as the expected future return. The securities’ expected returns then are calculated by means of the regression equations (10), the covariance matrix remains constant over the whole test period.

- A portfolio is purchased as proposed by any method, with or without the risk-free alternative; there are no short-sellings and no loans.
- The respective portfolios are rebalanced each month, thus accumulating respective surpluses or losses over the whole test period.
- We do not consider transaction costs.

Table 1 provides data for the control period: the prices of DAX, BASF, VOW and MAN in the upper block, followed in the second block by the monthly returns of the respective titles and the risk-free alternative. The DAX-returns appear in the third block as forecasts with a time lag of one month. The forecasts for BASF, VOW and MAN are calculated by means of the regression equations (10).

Return										
Realized (Price)	1-Jun-06	3-Jul-06	1-Aug-06	1-Sep-06	2-Oct-06	1-Nov-06	1-Dec-06	2-Jan-07	1-Feb-07	1-Mar-07
DAX	5707,5900	5712,6900	5596,7400	5876,5400	5999,4600	6291,9000	6241,1300	6681,1300	6851,2800	6640,2400
BASF (1)	63,2600	63,4500	62,4400	64,6600	63,7600	68,6700	69,1600	74,6900	75,6800	76,7200
VOW (2)	55,0000	55,2100	58,2000	63,2600	67,0000	77,5500	81,3200	85,1100	85,6800	93,4300
MAN (3)	57,4500	56,9500	56,9000	60,4700	69,7500	71,0500	71,0500	69,1000	81,6600	79,7500
Realized (%)		Jun-06	Jul-06	Aug-06	Sep-06	Oct-06	Nov-06	Dec-06	Jan-07	Feb-07
DAX		0,0893%	-2,0506%	4,8784%	2,0701%	4,7594%	-0,8102%	6,8126%	2,5148%	-3,1287%
BASF (1)		0,2999%	-1,6046%	3,4937%	-1,4017%	7,4186%	0,7110%	7,6924%	1,3168%	1,3649%
VOW (2)		0,3811%	5,2741%	8,3368%	5,7439%	14,6230%	4,7469%	4,5553%	0,6675%	8,6593%
MAN (3)		-0,8741%	-0,0878%	6,0852%	14,2770%	1,8466%	0,0000%	-2,7829%	16,7010%	-2,3668%
Risk-Free Rate (0)		0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%
Forecasted (%)			Jul-06	Aug-06	Sep-06	Oct-06	Nov-06	Dec-06	Jan-07	Feb-07
DAX			0,0893%	-2,0506%	4,8784%	2,0701%	4,7594%	-0,8102%	6,8126%	2,5148%
BASF (1)			0,6878%	-1,0063%	4,4793%	2,2560%	4,3850%	-0,0243%	6,0106%	2,6081%
VOW (2)			0,0618%	-1,6690%	3,9352%	1,6639%	3,8390%	-0,6658%	5,4997%	2,0236%
MAN (3)			1,4788%	-0,8782%	6,7538%	3,6606%	6,6227%	0,4880%	8,8843%	4,1504%
Risk-Free Rate (0)			0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%

Return										
Realized (Price)	2-Apr-07	2-May-07	4-Jun-07	2-Jul-07	1-Aug-07	3-Sep-07	1-Oct-07	1-Nov-07	3-Dec-07	2-Jan-08
DAX	6937,1700	7455,9300	7976,7900	7958,2400	7473,9300	7648,5800	7922,4200	7880,8500	7837,2600	7949,1100
BASF (1)	84,1000	87,2600	92,4500	96,0300	93,7400	96,5500	97,1500	93,2100	94,6600	100,3100
VOW (2)	111,9300	111,7200	115,0000	116,9600	134,4000	153,3100	161,2000	193,1200	161,7000	153,8800
MAN (3)	86,8000	101,4000	110,6800	107,2300	106,0500	107,2500	104,1800	123,6000	109,1500	110,9300
Realized (%)	Mar-07	Apr-07	May-07	Jun-07	Jul-07	Aug-07	Sep-07	Oct-07	Nov-07	Dec-07
DAX	4,3746%	7,2116%	6,7526%	-0,2328%	-6,2787%	2,3099%	3,5177%	-0,5261%	-0,5546%	1,4171%
BASF (1)	9,1844%	3,6886%	5,7776%	3,7993%	-2,4136%	2,9536%	0,6195%	-4,1401%	1,5437%	5,7974%
VOW (2)	18,0661%	-0,1878%	2,8936%	1,6900%	13,8988%	13,1642%	5,0184%	18,0666%	-17,7569%	-4,9570%
MAN (3)	8,4710%	15,5466%	8,7570%	-3,1667%	-1,1065%	1,1252%	-2,9042%	17,0930%	-12,4327%	1,6176%
Risk-Free Rate (0)	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%
Forecasted (%)	Mar-07	Apr-07	May-07	Jun-07	Jul-07	Aug-07	Sep-07	Oct-07	Nov-07	Dec-07
DAX	-3,1287%	4,3746%	7,2116%	6,7526%	-0,2328%	-6,2787%	2,3099%	3,5177%	-0,5261%	-0,5546%
BASF (1)	-1,8599%	4,0804%	6,3264%	5,9631%	0,4328%	-4,3537%	2,4458%	3,4020%	0,2006%	0,1780%
VOW (2)	-2,5411%	3,5278%	5,8224%	5,4512%	-0,1988%	-5,0888%	1,8578%	2,8347%	-0,4360%	-0,4591%
MAN (3)	-2,0658%	6,1989%	9,3238%	8,8183%	1,1240%	-5,5354%	3,9247%	5,2550%	0,8009%	0,7695%
Risk-Free Rate (0)	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%	0,4197%

Table 1: Prices, realized and predicted returns for July 06 until December 07

These data are the same for all portfolio selection methods. So the above assumptions, even if pretty restrictive, will not bias the comparison results.

As the Expert System Shell SPIRIT only accepts discrete rather than continuous variables, respective transformations are necessary. That is what the remainder of this subsection is about.

The reader is invited to go back to Subsection 2.3.3 and to repeat the variables DAX, BASF, VOW and MAN, their respective return classes and the corresponding utilities.

class	$r < -9$	$-9 \leq r < -6$	$-6 \leq r < -3$	$-3 \leq r < 0$
mean	-27.12	-7.5	-4.5	-1.5
name	*rmm*	*rm750*	*rm450*	*rm150*
class	$0 \leq r < 3$	$3 \leq r < 6$	$6 \leq r < 9$	$9 \leq r$
mean	1.5	4.5	7.5	19.53
name	*rp150*	*rp450*	*rp750*	*rpp*

Table 2: Classes of monthly returns in %

Table 2 shows the return classes and their means = utilities. For all means and all stocks the distributions R_{stock}(mean) are determined and again discretized. Figure 1 visualizes such conditioned discretized distributions for the selected DAX return class DAX=*rp*150, activated by a mouse click on the respective bar. Note that all these dependencies are informed to the shell SPIRIT via rules like those in Subsection 2.3.2.
That is what data and the test design are concerned with.

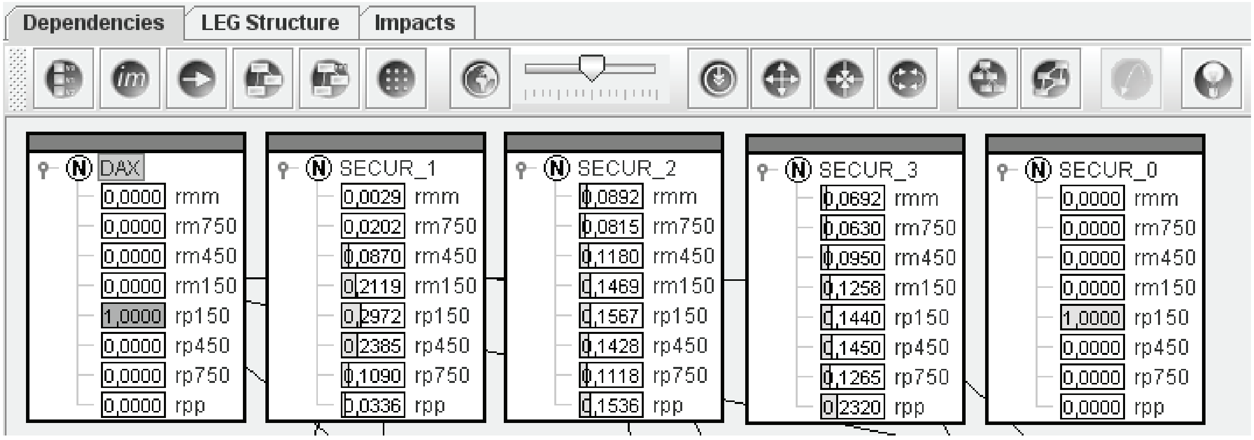

Figure 1: Discrete conditioned stock distribution for a selected DAX forecast

3.2 A first comparison of selection methods

Now, as the predicted returns of all securities, their correlations and probability distributions are available, the different selection methods can be applied. Respective portfolio selection proposals for each month and for all investor risk attitudes can be calculated, and the so determined portfolios can be evaluated by their corresponding monthly real returns. Before showing the accumulated return results in Subsection 3.3, however, we first relate on the portfolios for one special month so as to sensitize the reader to the different methods' behaviour.

Please verify from Table 1 the DAX naïve forecast for month June 07 to be 6.75 % and the respective forecasts of the securities to be BASF 5.96 %, VOW 5.45 %, MAN 8.82 %. The risk-free bond yields a moderate monthly 0.42 %. As the covariance matrix remains constant over the control period, the objective function of problem (2) in Subsection 2.2 now reads

$$\max Y = w_1 \cdot .0596 + w_2 \cdot .0545 + w_3 \cdot .0882 - \lambda \cdot \sqrt{(w_1, w_2, w_3)\mathbf{V}\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}}\text{, being } \mathbf{V} \text{ like in (11).}$$

The remaining optimization problems (3) to (5) are specified similarly. The following table shows numerical results for all methods and selected λ's.

		portfolio				
method	λ	w_0	w_1	w_2	w_3	realized returns
(3)	-	-	.70	.01	.29	1.76 %
(2)	.94	-	.71	.03	.26	1.91 %
(2)	.00	-	.00	.00	1.00	-3.17 %
(5)	-	.00	.70	.01	.29	1.76 %
(4)	.94	1.00	.00	.00	.00	.42 %
(4)	.00	.00	.00	.00	1.00	-3.17 %

Table 3: Portfolios and returns in % for June 07 and selected methods.

All results are obtained by LINGO calculations, www.lindo.com (2008), a professional software which solves linear and nonlinear optimization problems.

A major effort is taken to demonstrate the portfolio selection process as realized by SPIRIT. Once all rules are informed to the system, cf. Subsection 2.3.3, and once all rules are learned like in (7) the knowledge base is ready to support the investor in making good portfolio decisions. To inform the user about the actual status, the shell offers a so called dependency graph like in Figure 2.

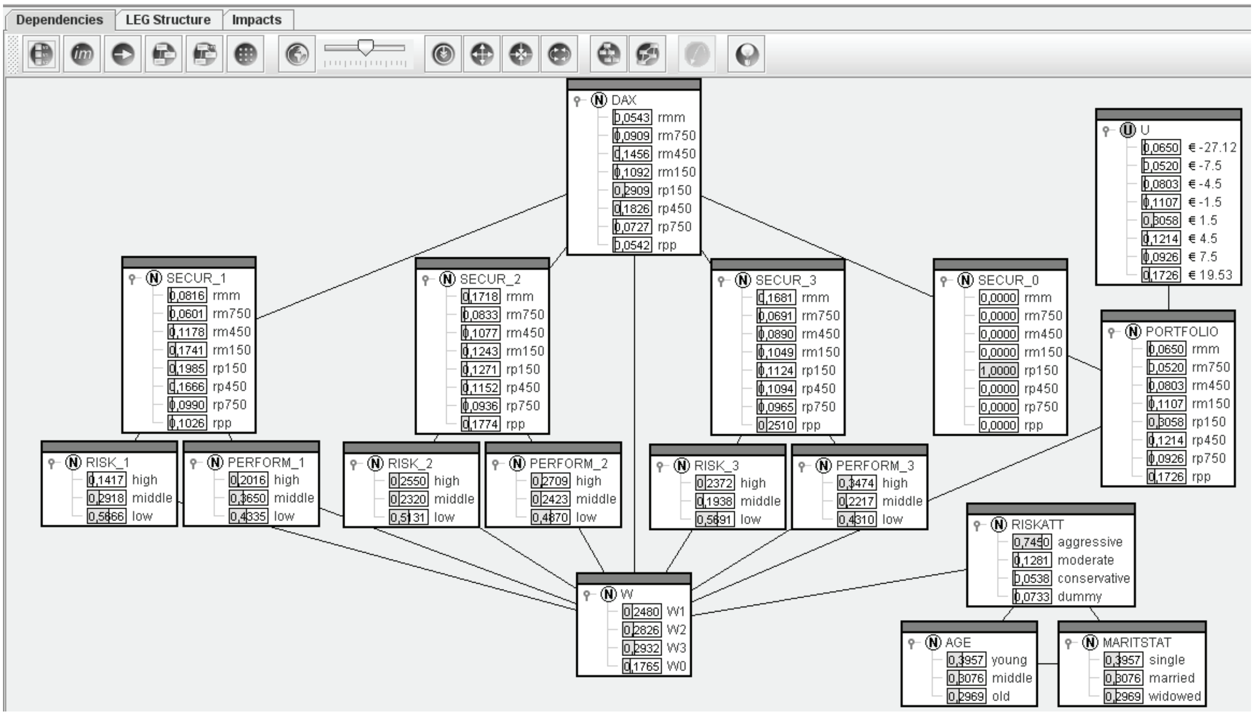

Figure 2: Dependency graph for the portfolio selection model.

The screenshot shows all involved variables with their respective attributes as nodes, connected by edges indicating stochastic dependencies. The graph is a Markov-net, for more details see Rödder et al. (2006). Each node furthermore shows the marginal distributions of the respective variables under P^*.

A mouse click on any attribute adapts P^* to a fact like in (8). If we want to compose a portfolio for a young and single investor whose risk attitude is very aggressive, and if now the DAX forecast is a good *rp750*, then the system proposes a 0 % risk-free application and almost equal investments in the stocks BASF (35 %) and VOW (35 %) as well as a slightly minor part of 31 % in MAN, cf. Figure 3.

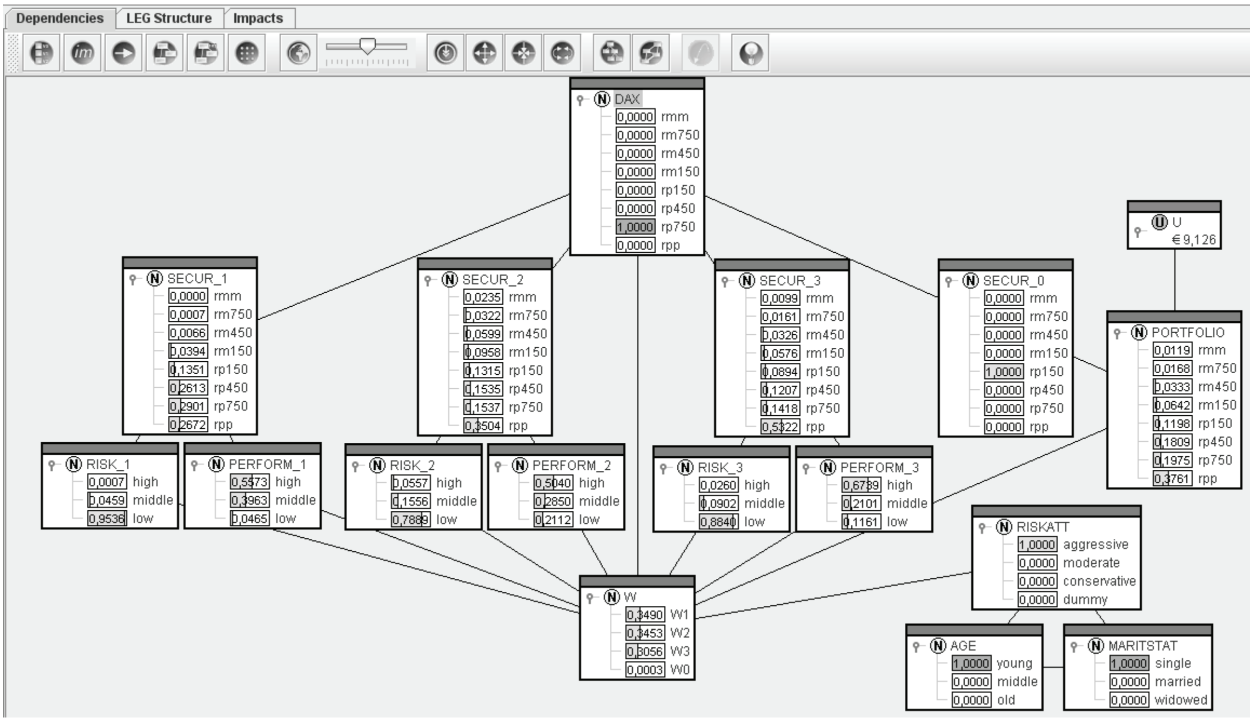

Figure 3: Portfolio selection by SPIRIT

The expected return with this strategy amounts to 9 %, see the U-variable. This value might be quite different from the later realized return which is a function of observed rather than of expected stock behaviour, of course. Please note the portfolio to be quite different from the one proposed in Table 3 for a risk accepting $\lambda = 0$.

If the same investor is confronted with a very bad DAX = *rmm* situation, he or she invests almost nothing in risky securities, but keeps 99 % in a risk-free bond. The expected monthly return now is just the one of the risk-free asset. The reader is invited to verify these results under www.xspirit.de, file: “Portfolio_BASF_VW_MAN.spirit”.

That is what a first comparison of the different selection methods for a single month was about, the next subsection evaluates the different methods by a 1½-years comparison.

3.3 Evaluation of Different Methods over a Control Period

As was mentioned before the comparison in this subsection consists of repetitive investments over 1½-years, in monthly varying portfolios rebalanced at the end of each period and accumulating returns or losses. Tables 4-9 show the portfolio weights, the realized monthly returns per security, the realized total returns per month and the accumulated returns in the last column.

For a good reason we start with the solution of equation (3) in Subsection 2.2. The total return over the test period is a moderate 37.4 %, see Table 4. Because of the relationship between equations (3) and (2), for monthly varying $\lambda = \Theta_t^*$ in (2) the portfolios are identical with the tangent portfolios of (3).

m	$\lambda = \Theta_t^*$	w_1	w_2	w_3	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,09605871	0,00000000	0,00000000	1,00000000	-1,6046%	5,2741%	-0,0878%	-0,0878%	0,99912165
Aug-06	-0,11773050	0,00000000	0,00000000	1,00000000	3,4937%	8,3368%	6,0852%	6,0852%	1,05992024
Sep-06	0,65477680	0,67756210	0,00000000	0,32243790	-1,4017%	5,7439%	14,2770%	3,6537%	1,09864682
Oct-06	0,31537550	0,54777300	0,00000000	0,45222700	7,4186%	14,6230%	1,8466%	4,8988%	1,15246768
Nov-06	0,64028190	0,67502840	0,00000000	0,32497160	0,7110%	4,7469%	0,0000%	0,4800%	1,15799908
Dec-06	0,00619275	0,00000000	0,00000000	1,00000000	7,6924%	4,5553%	-2,7829%	-2,7829%	1,12577298
Jan-07	0,89089250	0,69861990	0,01347124	0,28790880	1,3168%	0,6675%	16,7010%	5,7373%	1,19036158
Feb-07	0,36849560	0,58604560	0,00000000	0,41395440	1,3649%	8,6593%	-2,3668%	-0,1799%	1,18822055
Mar-07	-0,22544640	0,00000000	0,00000000	1,00000000	9,1844%	18,0661%	8,4710%	8,4710%	1,28887457
Apr-07	0,59345790	0,66593980	0,00000000	0,33406020	3,6886%	-0,1878%	15,5466%	7,6499%	1,38747186
May-07	0,93977540	0,69872430	0,02012058	0,28115510	5,7776%	2,8936%	8,7570%	6,5572%	1,47845163
Jun-07	0,88355340	0,69860330	0,01240425	0,28899250	3,7993%	1,6900%	-3,1667%	1,7600%	1,50447222
Jul-07	0,06387523	0,00000000	0,00000000	1,00000000	-2,4136%	13,8988%	-1,1065%	-1,1065%	1,48782466
Aug-07	-0,54014780	0,00000000	0,00000000	1,00000000	2,9536%	13,1642%	1,1252%	1,1252%	1,50456548
Sep-07	0,34396090	0,57007570	0,00000000	0,42992430	0,6195%	5,0184%	-2,9042%	-0,8954%	1,49109313
Oct-07	0,48943930	0,63898710	0,00000000	0,36101290	-4,1401%	18,0666%	17,0930%	3,5253%	1,54365903
Nov-07	0,03457517	0,00000000	0,00000000	1,00000000	1,5437%	-17,7569%	-12,4327%	-12,4327%	1,35173982
Dec-07	0,03172249	0,00000000	0,00000000	1,00000000	5,7974%	-4,9570%	1,6176%	1,6176%	1,37360595

Table 4: Optimal portfolios for (3) and for (2) with $\lambda = \Theta_t^*$

The reader hopefully agrees that such varying λ's in (2) are somewhat like a moderate risk attitude whereas $\lambda = \Theta_{max}^*$ represents a conservative strategy. Here Θ_{max}^* is the maximum of all ever observed Sharpe ratios.

m	$\lambda = \Theta^*_{\max}$	w_1	w_2	w_3	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,93977540	0,87867310	0,10368300	0,01764388	-1,6046%	5,2741%	-0,0878%	-0,8646%	0,99135360
Aug-06	0,93977540	0,89394180	0,10605820	0,00000000	3,4937%	8,3368%	6,0852%	4,0073%	1,03108031
Sep-06	0,93977540	0,76248890	0,05086727	0,18664380	-1,4017%	5,7439%	14,2770%	1,8881%	1,05054847
Oct-06	0,93977540	0,83216820	0,08305791	0,08477388	7,4186%	14,6230%	1,8466%	7,5447%	1,12980879
Nov-06	0,93977540	0,76557160	0,05232322	0,18210520	0,7110%	4,7469%	0,0000%	0,7927%	1,13876493
Dec-06	0,93977540	0,89165310	0,10834690	0,00000000	7,6924%	4,5553%	-2,7829%	7,3525%	1,22249217
Jan-07	0,93977540	0,71014610	0,02571429	0,26413960	1,3168%	0,6675%	16,7010%	5,3636%	1,28806234
Feb-07	0,93977540	0,82149080	0,07822413	0,10028510	1,3649%	8,6593%	-2,3668%	1,5612%	1,30817194
Mar-07	0,93977540	0,89593250	0,10406750	0,00000000	9,1844%	18,0661%	8,4710%	10,1087%	1,44041125
Apr-07	0,93977540	0,77544470	0,05696709	0,16758820	3,6886%	-0,1878%	15,5466%	5,4550%	1,51898584
May-07	0,93977540	0,69872430	0,02012058	0,28115510	5,7776%	2,8936%	8,7570%	6,5572%	1,61858929
Jun-07	0,93977540	0,71184150	0,02654141	0,26161710	3,7993%	1,6900%	-3,1667%	1,9209%	1,64968023
Jul-07	0,93977540	0,88611750	0,10691970	0,00696278	-2,4136%	13,8988%	-1,1065%	-0,6603%	1,63878662
Aug-07	0,93977540	0,90175620	0,09824384	0,00000000	2,9536%	13,1642%	1,1252%	3,9567%	1,70362892
Sep-07	0,93977540	0,82642460	0,08046235	0,09311308	0,6195%	5,0184%	-2,9042%	0,6454%	1,71462332
Oct-07	0,93977540	0,79697550	0,06698832	0,13603620	-4,1401%	18,0666%	17,0930%	0,2360%	1,71866900
Nov-07	0,93977540	0,89112900	0,10887100	0,00000000	1,5437%	-17,7569%	-12,4327%	-0,5576%	1,70908537
Dec-07	0,93977540	0,89118170	0,10881830	0,00000000	5,7974%	-4,9570%	1,6176%	4,6271%	1,78816673

Table 5: Conservative optimal portfolios for (2) with $\lambda = \Theta^*_{max}$

Table 5 shows a considerable accumulated return of nearly 79 % for the control period, even if the investor is conservative. One reason for this good result might be the avoidance of volatile stocks in favour of less volatile ones. Finally Table 6 shows the results for an aggressive, risk accepting $\lambda = 0$.

m	$\lambda = 0$	w_1	w_2	w_3	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,00000000	0,00000000	0,00000000	1,00000000	-1,6046%	5,2741%	-0,0878%	-0,0878%	0,99912165
Aug-06	0,00000000	0,00000000	0,00000000	1,00000000	3,4937%	8,3368%	6,0852%	6,0852%	1,05992024
Sep-06	0,00000000	0,00000000	0,00000000	1,00000000	-1,4017%	5,7439%	14,2770%	14,2770%	1,21124510
Oct-06	0,00000000	0,00000000	0,00000000	1,00000000	7,4186%	14,6230%	1,8466%	1,8466%	1,23361247
Nov-06	0,00000000	0,00000000	0,00000000	1,00000000	0,7110%	4,7469%	0,0000%	0,0000%	1,23361247
Dec-06	0,00000000	0,00000000	0,00000000	1,00000000	7,6924%	4,5553%	-2,7829%	-2,7829%	1,19928212
Jan-07	0,00000000	0,00000000	0,00000000	1,00000000	1,3168%	0,6675%	16,7010%	16,7010%	1,39957369
Feb-07	0,00000000	0,00000000	0,00000000	1,00000000	1,3649%	8,6593%	-2,3668%	-2,3668%	1,36644922
Mar-07	0,00000000	1,00000000	0,00000000	0,00000000	9,1844%	18,0661%	8,4710%	9,1844%	1,49194957
Apr-07	0,00000000	0,00000000	0,00000000	1,00000000	3,6886%	-0,1878%	15,5466%	15,5466%	1,72389770
May-07	0,00000000	0,00000000	0,00000000	1,00000000	5,7776%	2,8936%	8,7570%	8,7570%	1,87485953
Jun-07	0,00000000	0,00000000	0,00000000	1,00000000	3,7993%	1,6900%	-3,1667%	-3,1667%	1,81548818
Jul-07	0,00000000	0,00000000	0,00000000	1,00000000	-2,4136%	13,8988%	-1,1065%	-1,1065%	1,79539911
Aug-07	0,00000000	1,00000000	0,00000000	0,00000000	2,9536%	13,1642%	1,1252%	2,9536%	1,84842805
Sep-07	0,00000000	0,00000000	0,00000000	1,00000000	0,6195%	5,0184%	-2,9042%	-2,9042%	1,79474529
Oct-07	0,00000000	0,00000000	0,00000000	1,00000000	-4,1401%	18,0666%	17,0930%	17,0930%	2,10152177
Nov-07	0,00000000	0,00000000	0,00000000	1,00000000	1,5437%	-17,7569%	-12,4327%	-12,4327%	1,84024490
Dec-07	0,00000000	0,00000000	0,00000000	1,00000000	5,7974%	-4,9570%	1,6176%	1,6176%	1,87001324

Table 6: Optimal portfolios for (2) with $\lambda = 0$

Here each portfolio only consists of one security, namely the one with maximal expected return. This is totally different from the conservative strategy in Table 5, nevertheless both cases show an almost equally high performance.

The next tests are those for portfolios including the risk-free alternative. As the tables would give no new insights we put them in the Appendix, Tables A1, A2, A3. A1 shows the results for the modified Sharpe ratio (5) – and likewise for (4) with varying $\lambda = \Delta_t^*$, cf. subsection 2.2. A2 relates on portfolios for $\lambda = \Delta_{\max}^*$, the maximum of ever observed Sharpe ratios, and A3 shows portfolios for $\lambda = 0$. The respective total returns amount to 19.5 % in A1, 14.4 % in A2, 58.8 % in A3.

These accumulated returns remain significantly below those in Tables 4-6. The option to invest in a risk-free but little profitable bond often causes unnecessary cautiousness.

The following 3 tables show the investment behaviour of the SPIRIT models. We again consider first a moderate, than a conservative and finally an aggressive investor.

m	w_0	w_1	w_2	w_3	*r-real_0*	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,07828338	0,29231893	0,29147819	0,33791950	0,4197%	-1,6046%	5,2741%	-0,0878%	1,0714%	1,01071412
Aug-06	0,26937674	0,20946502	0,24251101	0,27864724	0,4197%	3,4937%	8,3368%	6,0852%	4,5623%	1,05682549
Sep-06	0,01460021	0,33450637	0,31730742	0,33358601	0,4197%	-1,4017%	5,7439%	14,2770%	6,1225%	1,12152924
Oct-06	0,07828338	0,29231893	0,29147819	0,33791950	0,4197%	7,4186%	14,6230%	1,8466%	7,0878%	1,20102074
Nov-06	0,01460021	0,33450637	0,31730742	0,33358601	0,4197%	0,7110%	4,7469%	0,0000%	1,7502%	1,22204100
Dec-06	0,26937674	0,20946502	0,24251101	0,27864724	0,4197%	7,6924%	4,5553%	-2,7829%	2,0536%	1,24713678
Jan-07	0,00187108	0,34840388	0,34468785	0,30503719	0,4197%	1,3168%	0,6675%	16,7010%	5,7840%	1,31927169
Feb-07	0,07828338	0,29231893	0,29147819	0,33791950	0,4197%	1,3649%	8,6593%	-2,3668%	2,1561%	1,34771593
Mar-07	0,55200108	0,10600463	0,16656882	0,17542547	0,4197%	9,1844%	18,0661%	8,4710%	5,7006%	1,42454336
Apr-07	0,01460021	0,33450637	0,31730742	0,33358601	0,4197%	3,6886%	-0,1878%	15,5466%	6,3665%	1,51523735
May-07	0,00187108	0,34840388	0,34468785	0,30503719	0,4197%	5,7776%	2,8936%	8,7570%	5,6823%	1,60133815
Jun-07	0,00187108	0,34840388	0,34468785	0,30503719	0,4197%	3,7993%	1,6900%	-3,1667%	0,9410%	1,61640704
Jul-07	0,26937674	0,20946502	0,24251101	0,27864724	0,4197%	-2,4136%	13,8988%	-1,1065%	2,6698%	1,65956186
Aug-07	0,77239007	0,03714241	0,09884670	0,09162082	0,4197%	2,9536%	13,1642%	1,1252%	1,8382%	1,69006841
Sep-07	0,07828338	0,29231893	0,29147819	0,33791950	0,4197%	0,6195%	5,0184%	-2,9042%	0,6953%	1,70181954
Oct-07	0,01460021	0,33450637	0,31730742	0,33358601	0,4197%	-4,1401%	18,0666%	17,0930%	10,0559%	1,87295272
Nov-07	0,26937674	0,20946502	0,24251101	0,27864724	0,4197%	1,5437%	-17,7569%	-12,4327%	-7,3342%	1,73558690
Dec-07	0,26937674	0,20946502	0,24251101	0,27864724	0,4197%	5,7974%	-4,9570%	1,6176%	0,5760%	1,74558467

Table 7: Optimal MaxEnt-portfolios for a moderate investor

m	w_0	w_1	w_2	w_3	*r-real_0*	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,46722208	0,16896849	0,16848253	0,19532690	0,4197%	-1,6046%	5,2741%	-0,0878%	0,7964%	1,00796424
Aug-06	0,79196577	0,05964208	0,06905144	0,07934070	0,4197%	3,4937%	8,3368%	6,0852%	1,5993%	1,02408419
Sep-06	0,13268673	0,29442041	0,27928252	0,29361034	0,4197%	-1,4017%	5,7439%	14,2770%	5,4391%	1,07978485
Oct-06	0,46722208	0,16896849	0,16848253	0,19532690	0,4197%	7,4186%	14,6230%	1,8466%	4,2741%	1,12593540
Nov-06	0,13268673	0,29442041	0,27928252	0,29361034	0,4197%	0,7110%	4,7469%	0,0000%	1,5908%	1,14384636
Dec-06	0,79196577	0,05964208	0,06905144	0,07934070	0,4197%	7,6924%	4,5553%	-2,7829%	0,8850%	1,15396888
Jan-07	0,01898824	0,34242902	0,33877671	0,29980604	0,4197%	1,3168%	0,6675%	16,7010%	5,6920%	1,21965334
Feb-07	0,46722208	0,16896849	0,16848253	0,19532690	0,4197%	1,3649%	8,6593%	-2,3668%	1,4234%	1,23701362
Mar-07	0,92712628	0,01724324	0,02709491	0,02853557	0,4197%	9,1844%	18,0661%	8,4710%	1,2787%	1,25283185
Apr-07	0,13268673	0,29442041	0,27928252	0,29361034	0,4197%	3,6886%	-0,1878%	15,5466%	5,6539%	1,32366558
May-07	0,01898824	0,34242902	0,33877671	0,29980604	0,4197%	5,7776%	2,8936%	8,7570%	5,5921%	1,39768604
Jun-07	0,01898824	0,34242902	0,33877671	0,29980604	0,4197%	3,7993%	1,6900%	-3,1667%	0,9321%	1,41071358
Jul-07	0,79196577	0,05964208	0,06905144	0,07934070	0,4197%	-2,4136%	13,8988%	-1,1065%	1,0604%	1,42567292
Aug-07	0,97225221	0,00452801	0,01205034	0,01116944	0,4197%	2,9536%	13,1642%	1,1252%	0,5927%	1,43412238
Sep-07	0,46722208	0,16896849	0,16848253	0,19532690	0,4197%	0,6195%	5,0184%	-2,9042%	0,5790%	1,44242628
Oct-07	0,13268673	0,29442041	0,27928252	0,29361034	0,4197%	-4,1401%	18,0666%	17,0930%	8,9011%	1,57081859
Nov-07	0,79196577	0,05964208	0,06905144	0,07934070	0,4197%	1,5437%	-17,7569%	-12,4327%	-1,7881%	1,54273113
Dec-07	0,79196577	0,05964208	0,06905144	0,07934070	0,4197%	5,7974%	-4,9570%	1,6176%	0,4642%	1,54989316

Table 8: Optimal MaxEnt-portfolios for a conservative investor

m	w_0	w_1	w_2	w_3	*r-real_0*	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,00990464	0,31400498	0,31310188	0,36298850	0,4197%	-1,6046%	5,2741%	-0,0878%	1,1198%	1,01119758
Aug-06	0,04161923	0,27476164	0,31810906	0,36551006	0,4197%	3,4937%	8,3368%	6,0852%	5,8536%	1,07038909
Sep-06	0,00174212	0,33887120	0,32144784	0,33793884	0,4197%	-1,4017%	5,7439%	14,2770%	6,1969%	1,13671977
Oct-06	0,00990464	0,31400498	0,31310188	0,36298850	0,4197%	7,4186%	14,6230%	1,8466%	7,5825%	1,22291103
Nov-06	0,00174212	0,33887120	0,32144784	0,33793884	0,4197%	0,7110%	4,7469%	0,0000%	1,7676%	1,24452673
Dec-06	0,04161923	0,27476164	0,31810906	0,36551006	0,4197%	7,6924%	4,5553%	-2,7829%	2,5629%	1,27642294
Jan-07	0,00022075	0,34897994	0,34525776	0,30554155	0,4197%	1,3168%	0,6675%	16,7010%	5,7929%	1,35036498
Feb-07	0,00990464	0,31400498	0,31310188	0,36298850	0,4197%	1,3649%	8,6593%	-2,3668%	2,2849%	1,38121903
Mar-07	0,12673547	0,20663015	0,32468525	0,34194913	0,4197%	9,1844%	18,0661%	8,4710%	10,7134%	1,52919485
Apr-07	0,00174212	0,33887120	0,32144784	0,33793884	0,4197%	3,6886%	-0,1878%	15,5466%	6,4441%	1,62773812
May-07	0,00022075	0,34897994	0,34525776	0,30554155	0,4197%	5,7776%	2,8936%	8,7570%	5,6910%	1,72037323
Jun-07	0,00022075	0,34897994	0,34525776	0,30554155	0,4197%	3,7993%	1,6900%	-3,1667%	0,9419%	1,73657709
Jul-07	0,04161923	0,27476164	0,31810906	0,36551006	0,4197%	-2,4136%	13,8988%	-1,1065%	3,3712%	1,79512078
Aug-07	0,28556153	0,11658529	0,31026716	0,28758603	0,4197%	2,9536%	13,1642%	1,1252%	4,8722%	1,88258268
Sep-07	0,00990464	0,31400498	0,31310188	0,36298850	0,4197%	0,6195%	5,0184%	-2,9042%	0,7157%	1,89605723
Oct-07	0,00174212	0,33887120	0,32144784	0,33793884	0,4197%	-4,1401%	18,0666%	17,0930%	10,1816%	2,08910683
Nov-07	0,04161923	0,27476164	0,31810906	0,36551006	0,4197%	1,5437%	-17,7569%	-12,4327%	-9,7513%	1,88539136
Dec-07	0,04161923	0,27476164	0,31810906	0,36551006	0,4197%	5,7974%	-4,9570%	1,6176%	0,6248%	1,89717074

Table 9: Optimal MaxEnt- portfolios for an aggressive investor

The new approach shows a similar tendency as the other methods do: the more aggressive the less risk-free investments. On the other hand we can not repeat the above criticism that "the option to invest in a risk-free but little profitable bond often causes unnecessary cautiousness". The system controls the weights (w_0, w_1, w_2, w_3) in a very competent way and realizes excellent cumulative returns. The following table gives an overview over all results.

risk-attitude / **method**	moderate	conservative	aggressive
Markowitz excl. w_0	37.4 %	78.9 %	87.0 %
Markowitz incl. w_0	19.5 %	14.4 %	58.8 %
SPIRIT	74.6 %	55.0 %	89.7 %

Table 10: Cumulative returns, all methods and all risk attitudes

The last two lines of Table 10 compare the cumulative returns, permitting a risk-free investment. SPIRIT fully dominates Markowitz (and Sharpe). The first line shows results for the little real situation that all money must be applied in risky stocks, no risk-free bond allowed. Nevertheless in two out of three cases SPIRIT shows better returns and loses only once. The best ever managed return amounts to almost 90 % for an aggressive SPIRIT strategy.

Scientific honesty, however, demands the remark that all results are a mere 1½-years snapshot under very restrictive assumptions. Even though might portfolio selection under MaxEnt or MinREnt, respectively, turn out to be a guiding instrument in the future.

4. Resume and Future Research

In the present contribution the classical portfolio theory by Markowitz and Sharpe is reviewed and the therein manifest MV-philosophy once again criticized, as it equally punishes positive and negative risks. The respective parametric and fractional programming problems also are repeated, and relations between the respective models are shown so as to facilitate further comparisons. Then a new method is presented which determines the portfolio weights by a rule based inference mechanism under Maximum Entropy and Minimum Relative Entropy, respectively. This inference mechanism respects all rules provided by an expert, and does not add any not intended dependencies between the involved variables. The classical expected returns and the variance of a portfolio now appear as the securities' performance chances and their short-fall risks, no numerical calculations of expectations or covariances involved. The method leads to a model including 16 variables and 432 rules. Equipped with such knowledge the new approach for any risk attitude of the investor and for any index forecast like that of the DAX, proposes a portfolio keeping an adequate portion of the investment capital in a risk-free alternative. The model is realized in the expert system Shell SPIRIT and then compared to classical Markowitz and Sharpe approaches. This is done by a simulation of BASF, Volkswagen and MAN stocks during the 1½-years period July 06 to December 07.

For a naïve forecast of the securities' returns and for the usual MV concept the classical methods propose respective portfolios. The new method takes into consideration the MinREnt inference mechanism instead. In a short-term analysis all portfolios are rebalanced each month and the returns are accumulated over the whole control period – no transaction costs considered.

The new method performs better than any of the classical approaches, with one exception.

Entropy-driven portfolio selection is a young field and so there remains a lot of work to do. DAX and the titles' monthly returns, risk attributes of the investor, risk and performance categories – all these things must be modelled as discrete variables in SPIRIT. Entropy and Relative Entropy might be a control concept even in the presence of continuous rather than of discrete variables. A first research on that subject shows promising results, cf. Singer (2008).

There are recent contributions on downside risk approaches, like that of Grootveld & Hallerbach (1999), e.g. To compete with such approaches is one principal challenge for the new method.

Appendix

m	$\lambda = \Delta_t^*$	w_0	w_1	w_2	w_3	*r-real_0*	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,09605871	0,00000000	0,00000000	0,00000000	0,99999960	0,4197%	-1,6046%	5,2741%	-0,0878%	-0,0878%	0,99912165
Aug-06	0,00000000	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	3,4937%	8,3368%	6,0852%	0,4197%	1,00331534
Sep-06	0,65477680	0,00000000	0,67756230	0,00000000	0,32243770	0,4197%	-1,4017%	5,7439%	14,2770%	3,6537%	1,03997370
Oct-06	0,31537550	0,00000000	0,54777520	0,00000000	0,45222480	0,4197%	7,4186%	14,6230%	1,8466%	4,8988%	1,09092040
Nov-06	0,64028190	0,00000000	0,67502850	0,00000000	0,32497150	0,4197%	0,7110%	4,7469%	0,0000%	0,4800%	1,09615639
Dec-06	0,00619275	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	7,6924%	4,5553%	-2,7829%	-2,7829%	1,06565132
Jan-07	0,89089250	0,00000000	0,69862020	0,01347097	0,28790890	0,4197%	1,3168%	0,6675%	16,7010%	5,7373%	1,12679060
Feb-07	0,36849560	0,00000000	0,58604740	0,00000000	0,41395260	0,4197%	1,3649%	8,6593%	-2,3668%	-0,1799%	1,12476399
Mar-07	0,00000000	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	9,1844%	18,0661%	8,4710%	0,4197%	1,12948504
Apr-07	0,59345790	0,00000000	0,66593990	0,00000000	0,33406010	0,4197%	3,6886%	-0,1878%	15,5466%	7,6499%	1,21588922
May-07	0,93977540	0,00000000	0,69872380	0,02012083	0,28115530	0,4197%	5,7776%	2,8936%	8,7570%	6,5572%	1,29561791
Jun-07	0,88355340	0,00000000	0,69860210	0,01240472	0,28899320	0,4197%	3,7993%	1,6900%	-3,1667%	1,7600%	1,31842057
Jul-07	0,06387523	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	-2,4136%	13,8988%	-1,1065%	-1,1065%	1,30383175
Aug-07	0,00000000	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	2,9536%	13,1642%	1,1252%	0,4197%	1,30930442
Sep-07	0,34396090	0,00000000	0,57007700	0,00000000	0,42992300	0,4197%	0,6195%	5,0184%	-2,9042%	-0,8954%	1,29758055
Oct-07	0,48943930	0,00000000	0,63898830	0,00000000	0,36101170	0,4197%	-4,1401%	18,0666%	17,0930%	3,5253%	1,34332417
Nov-07	0,03457517	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	1,5437%	-17,7569%	-12,4327%	-12,4327%	1,17631208
Dec-07	0,03172249	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	5,7974%	-4,9570%	1,6176%	1,6176%	1,19534045

A1: Optimal portfolios for (5) and for (4) with $\lambda = \Delta^*_t$

m	$\lambda = \Delta^*_{max}$	w_0	w_1	w_2	w_3	*r-real_0*	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,93977540	1,00000000	0,00000000	0,00000000	0,00000003	0,4197%	-1,6046%	5,2741%	-0,0878%	0,4197%	1,00419737
Aug-06	0,93977540	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	3,4937%	8,3368%	6,0852%	0,4197%	1,00841237
Sep-06	0,93977540	0,99999970	0,00000023	0,00000000	0,00000011	0,4197%	-1,4017%	5,7439%	14,2770%	0,4197%	1,01264506
Oct-06	0,93977540	0,99999980	0,00000008	0,00000000	0,00000007	0,4197%	7,4186%	14,6230%	1,8466%	0,4197%	1,01689552
Nov-06	0,93977540	0,99999970	0,00000022	0,00000000	0,00000011	0,4197%	0,7110%	4,7469%	0,0000%	0,4197%	1,02116381
Dec-06	0,93977540	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	7,6924%	4,5553%	-2,7829%	0,4197%	1,02545002
Jan-07	0,93977540	0,99999950	0,00000033	0,00000001	0,00000013	0,4197%	1,3168%	0,6675%	16,7010%	0,4197%	1,02975424
Feb-07	0,93977540	0,99999980	0,00000011	0,00000000	0,00000007	0,4197%	1,3649%	8,6593%	-2,3668%	0,4197%	1,03407650
Mar-07	0,93977540	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	9,1844%	18,0661%	8,4710%	0,4197%	1,03841691
Apr-07	0,93977540	0,99999970	0,00000020	0,00000000	0,00000010	0,4197%	3,6886%	-0,1878%	15,5466%	0,4197%	1,04277556
May-07	0,93977540	0,00000000	0,69872270	0,02012269	0,28115460	0,4197%	5,7776%	2,8936%	8,7570%	6,5572%	1,11115271
Jun-07	0,93977540	0,99999950	0,00000032	0,00000001	0,00000013	0,4197%	3,7993%	1,6900%	-3,1667%	0,4197%	1,11581664
Jul-07	0,93977540	1,00000000	0,00000000	0,00000000	0,00000002	0,4197%	-2,4136%	13,8988%	-1,1065%	0,4197%	1,12050014
Aug-07	0,93977540	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	2,9536%	13,1642%	1,1252%	0,4197%	1,12520330
Sep-07	0,93977540	0,99999980	0,00000010	0,00000000	0,00000007	0,4197%	0,6195%	5,0184%	-2,9042%	0,4197%	1,12992619
Oct-07	0,93977540	0,99999980	0,00000016	0,00000000	0,00000009	0,4197%	-4,1401%	18,0666%	17,0930%	0,4197%	1,13466893
Nov-07	0,93977540	1,00000000	0,00000000	0,00000000	0,00000001	0,4197%	1,5437%	-17,7569%	-12,4327%	0,4197%	1,13943156
Dec-07	0,93977540	1,00000000	0,00000000	0,00000000	0,00000001	0,4197%	5,7974%	-4,9570%	1,6176%	0,4197%	1,14421418

A2: Optimal portfolios for (4) and for $\lambda = \Delta^*_{max}$

m	$\lambda = 0$	w_0	w_1	w_2	w_3	*r-real_0*	*r-real_1*	*r-real_2*	*r-real_3*	*Sum r-real*w*	*Prod 1+r-real_t*
Jul-06	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	-1,6046%	5,2741%	-0,0878%	-0,0878%	0,99912165
Aug-06	0,00000000	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	3,4937%	8,3368%	6,0852%	0,4197%	1,00331534
Sep-06	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	-1,4017%	5,7439%	14,2770%	14,2770%	1,14655872
Oct-06	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	7,4186%	14,6230%	1,8466%	1,8466%	1,16773157
Nov-06	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	0,7110%	4,7469%	0,0000%	0,0000%	1,16773157
Dec-06	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	7,6924%	4,5553%	-2,7829%	-2,7829%	1,13523462
Jan-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	1,3168%	0,6675%	16,7010%	16,7010%	1,32482965
Feb-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	1,3649%	8,6593%	-2,3668%	-2,3668%	1,29347419
Mar-07	0,00000000	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	9,1844%	18,0661%	8,4710%	0,4197%	1,29890338
Apr-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	3,6886%	-0,1878%	15,5466%	15,5466%	1,50083930
May-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	5,7776%	2,8936%	8,7570%	8,7570%	1,63226790
Jun-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	3,7993%	1,6900%	-3,1667%	-3,1667%	1,58057872
Jul-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	-2,4136%	13,8988%	-1,1065%	-1,1065%	1,56308901
Aug-07	0,00000000	1,00000000	0,00000000	0,00000000	0,00000000	0,4197%	2,9536%	13,1642%	1,1252%	0,4197%	1,56964988
Sep-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	0,6195%	5,0184%	-2,9042%	-2,9042%	1,52406351
Oct-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	-4,1401%	18,0666%	17,0930%	17,0930%	1,78457225
Nov-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	1,5437%	-17,7569%	-12,4327%	-12,4327%	1,56270091
Dec-07	0,00000000	0,00000000	0,00000000	0,00000000	1,00000000	0,4197%	5,7974%	-4,9570%	1,6176%	1,6176%	1,58797961

A3: Optimal portfolios for (4) and for $\lambda = 0$

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