FernUniversität in Hagen

Stockkeeping and controlling under game theoretic aspects

Günter Fandel and Jan Trockel

Discussion paper No. 420

February 2008

Diskussionsbeiträge der Fakultät für Wirtschaftswissenschaft
der FernUniversität in Hagen

Herausgegeben vom Dekan der Fakultät

Stockkeeping and controlling under game theoretic aspects

Günter Fandel [1], Jan Trockel [2]

[1] Department of Economics and Business Administration, FernUniversität in Hagen, Universitätsstraße 41 / ESG, 58084 Hagen, Germany
guenter.fandel@fernuni-hagen.de
[2] Department of Economics and Business Administration, FernUniversität in Hagen, Universitätsstraße 41 / ESG, 58084 Hagen, Germany
jan.trockel@fernuni-hagen.de

Abstract

The stock level in industrial companies is frequently subject of critical discussions. Material managers tend towards high stock levels to ensure delivery and operational readiness. In contrast, controllers demand lower stock levels to minimize the costs of capital commitment. This decision conflict – based on lateral perception – can be modelled using an approach of game theory and it can be analysed in view of decision theory. This is the central object of this article. The consequences of decisions of the material manager and the controller in a company will be analysed if their actions result in different payoffs. Both the warehouseman and the controller each have two different alternatives to choose their own behaviour from with a specific probability. The material manager can select a low stock level at the risk of shortfalls or he can select a high stock level to ensurce delivery disposition. The controller can check the economic efficiency of the stock level on a low or a high audit level. With respect to the different strategy conditions and the respective payoffs we show the existence of non-cooperative Nash equilibria in dependence of specific probabilities by which the players choose their strategies. Following these actions the top management analyses how far the players have pursued the economic efficiency of the company and how far the company was damaged because of the choice of their actions. This damage can be exposed by the top management with a specific probability. Additionally, the analysis will provide the management with informations, how the payoffs should be specified in order to reduce the probability of bad stockkeeping and bad auditing.

Keywords: stockkeeping, auditing, non-cooperative game theory

1 Introduction

Procurement and stockkeeping are fundamental divisions of the operative value added. High costs can be the result of frequent orders and high stockkeeping in these divisions [12; 22]. Based on game theoretic analysis it will be examined in this article, how the interaction between the material manager and the controller of the company should be arranged so that the procurement costs are kept to a minimum. In this context it will be particularly discussed in how far the co-operation between the material manager and the controller can result in a bad planning decision. In order to avoid this situation, it is necessary to let the controller be monitored by the management. If those decisions result in high costs or profit setbacks, it has to be checked under which conditions this decision-making leads to a Nash equilibrium and under which conditions this will not be detected by the management. Because of these observations it can be concluded that the players´ payoffs of the game have to be configured in a way that good planner's decisions, being Nash equilibria of the non-cooperative game, will be generated ex-ante.

2 Basics and Approach

In the following it will be assumed that no contract exists between the material manager and the controller. Both players have their own formal agreement to the company. In the literature it is generally assumed that the controller behaves in terms of the company. He maximizes corporate success or minimizes costs [9]. This circumstance must sometimes be relativised, because controllers are also part of the company's system with their own needs and preferences. So the management cannot trust them with a proper inspection and a truthful reporting. There will occur conflicts of interests or problems of information if the players' intention do not correspond anymore [7]. On the basis of the bilateral relations between procurement and controlling it must be analysed, how one player's personal preferences affect the other player's behaviour. In the focus of this game theoretic analysis is the risk that faulty disposition and bad controlling in a company result in higher costs and that this will not be detected by the management. This article differs insofar from the already existing considerations of agency models, that there is no discussion about maximizing the company's profit considering the players' contracts and incentives, but instead there is an analysis of the behaviour between procurement and controlling.

Literature, which analyses inspections in economics, is already available: [5], [8], [4] and [10]. The "inspection game" provides a basis for the subsequent analysis, which was first formulated by [6] and which sampled as a decision concept in the domain of nuclear weapon control by [13] and [14; 15]. Thereby a contract between two parties exists: the inspector and the inspectee. This game provided a basis for [5], who analysed – based on the "inspection game" as a game theoretic model –, how an accountant and a businessman, alternatively an insurant and insurance, act among each other. [23] and [1; 2] take up this analysis and enhance it with problems of material accountancy and data verification.

In this article the interaction between the controller (inspector) and the material manager (inspectee) is not configurated vertically but laterally – this means that both players are on a par with each other and have their own scope for disposition – and the model "inspection game" is amplified by the possibility of monitoring the activities by dint of the management. Primarily the behaviour patterns of the two players, material manager and controller, are relevant, because they can lead to a procurement decision different from the optimal one. Thereby it must be defined, when a procurement decision seriously differs from the cost minimum. Research conclusions of [21] are integrated into the considered game theoretic model as follows. An incorrect lot size planning shall be existent if the cost difference $\Delta\hat{K}$ occurring when the material manager uses an economic order quantity differing from the optimal one, exceeds the amount ε (see figure 1). Controlling the tolerance is the controller's job. This task should be solved by modelling a modified "inspection game". Additionaly it has to be clarified to which negative effects a controller can be subjected to if he does not detect the fault of the inspectee because of a low inspection level.

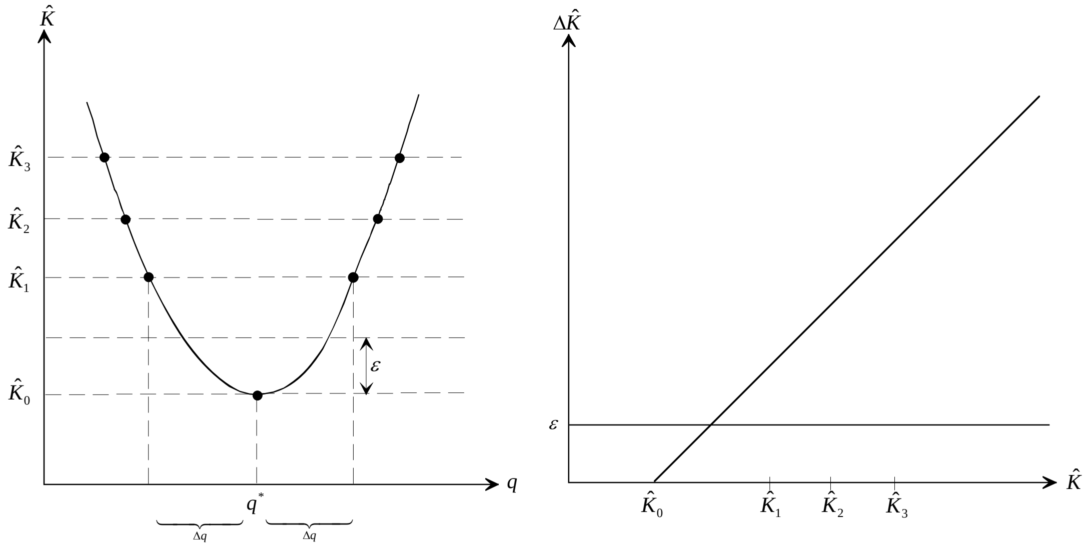

Figure 1: Increase of the cost by deviation of q;*
transformation of Δq into $\Delta\hat{K}$

3 Description of the players' activities

The reported modification of the "inspection game" shows that the controller in the role of the inspector generally checks the decision's optimality of the material manager. He can audit on a high level (h) or a low level (nh). The material manager (inspectee) can make a methodically established ordering decision (m) or he can make a decision by acting on instinct (nm). If he works methodically correct, he will realise a q near q^*. So the cost deviation can be disregarded [21]. If he acts on instinct, the inspectee benefits from investing the time left in leisure. He will gain an additional bonus. If his careless behaviour is detected because of a high audit by the controller, the material manager will be punished by the management (reputation deficit or disprofit). Otherwise the controller will get a bonus due to his successful auditing. If the inspector certifies a good behaviour after auditing on a high level, the inspectee will get an additional bonus by the management [19]. The payoffs of the two players can be taken from the subsequent bimatrix. If $K < B_C$ and $B_D > L - S$, a Nash equilibrium cannot be found in pure strategies. By using mixed strategies [18; 24] a Nash equilibrium can be obtained [3; 16; 17] for the probabilities p_m^* and p_h^* of planning methodically and on a high audit level.

Generally a mixed-strategy profile s^* will be a Nash equilibrium if, for all players *i*,

$$\pi_i\left(s_i^*, s_{-i}\right) \geq \pi_i\left(s_i, s_{-i}\right) \qquad \forall\, s_i \in S_i \setminus \left\{s_i^*\right\}.$$

Thereby π_i indicates the payoff, S_i denotes the set of all possible strategies of player *i* [17; 18], and s_{-i} indicates the strategies of the residual players.

controller / material manager	high auditing level (h) p_h	low auditing level (nh) $(1-p_h)$
methodically established decision (m) p_m	$Z+B_D$ / $V-K$	$Z+B_D$ / V
decision on instinct (nm) $(1-p_m)$	$Z-S+L$ / $V+B_C-K$	$Z+B_D+L$ / V

Figure 2: Modified „inspection game" between controller and material manager (*in dependence on* [2; 4])

Furthermore the circumstances of the case may be characterized by the subsequent symbols.

For the material manager:

Z *basic salary*,

S *punishment* if the insufficient decision is detected (expressed by reputation or material losses),

B_D *bonus* if the planning action of the material manager is certified as good work without complaints of the controller,

L *leisure profit*, which the material manager will get if he does not decide methodically,

p_m *probability* that the material manager assesses the optimal lot size methodically correct.

For the controller:

V *basic salary*,

K *additional costs* because of a high auditing level,

B_C *bonus* if the controller detects insufficient planning of the material manager,

p_h *probability* that the controller applies a high auditing level.

It is assumed that bad planning decisions of the material manager can be discovered with the same probability, with which the controller audits the planner's behaviour on a high level.
In this 2x2-game a Nash equilibrium exists in mixed strategies if and only if

$$p_m^* = \frac{B_C - K}{B_C} \text{ and } p_h^* = \frac{L}{S + B_D}.$$

These conditions will not be discussed any further, because the attention will be turned to the analysis now, how far the controller will be punished if he does not audit correctly. These facts are not included in the upper game because of the independent structure. Only if the results of the controller´s deviation analysis are inspected by the top management or a similar authority, the inspector will get a penalty. In the literature, for example [4], it is described that the inspector will be penalisied and so experience a monetary or reputational loss (punishment) if he does not detect the misdemeanour of the material manager. This economic interaction of two independent players cannot be analysed until a third party is introduced into the game.

4 Further modification of the modified "inspection game"

In the following the "inspection game" will be modified to include lateral conflict management [11] between the controller and the material manager induced by a higher ranking authority, where the controller can also suffer a loss S triggered by the material manager's malpractice. This loss should be equal to the loss of the material manager. It will occur if the higher-ranking management detects the malpractice of both players (strategy sequence (nm, nh)) with a probability p_a (strategy (a) of the management). This extension is demonstrated in figure 3. As a result of wrong evidence about a reportedly optimal planning of the material manager the controller can be prosecuted if he does not detect the bad disposition in consequence of a low auditing level. His result will be a lower reputation level or a disprofit. The game is modified from primarily four to five end nodes. Thereby a fault can only be revealed by the top management if the material manager as well as the controller does not act in terms of the company. The consequence will be a cost increase for the company. The controller will not behave in terms of the management if he has the chance to choose another strategy to get a higher payoff instead of being a correct controller for the company's profit maximizing strategy. But if he detects the wrong manager's behaviour, he can not change the consequences ex-post, but the top management can honour the circumstances positively.

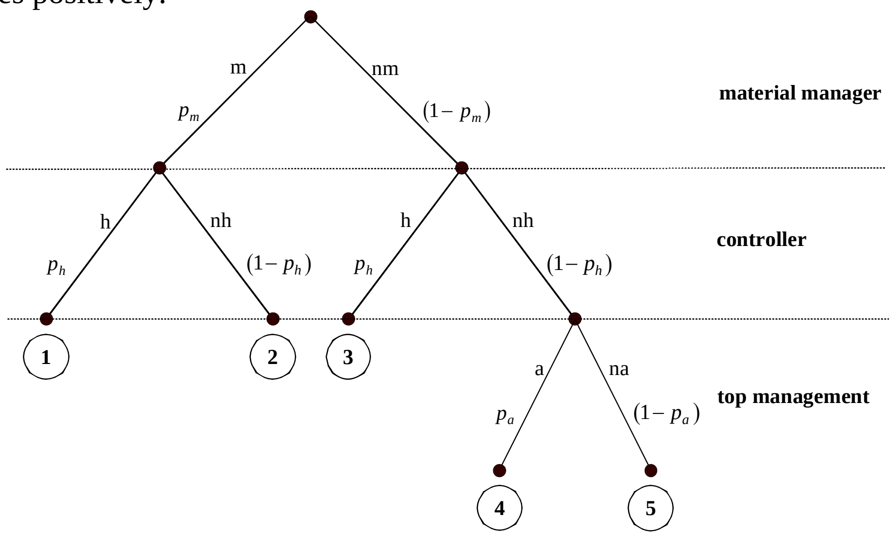

Figure 3: Further modification of the modified "inspection game" by introducing the top management

In figure 3 there is only one additional bifurcation in the right branch. For this proceeding obvious causes exist. If the material manager does his work methodically correct, the top management cannot detect wrong behaviour. A bifurcation at the nodes (m, h) and (m, nh) to differentiate between detection or no detection is useless. This is the reason why there are

only two end nodes in the left branch of the modified game, which have to be estimated. If the planner's work is not correct, this can be detected by the controller in case of a high audit level or in case of a controller's low audit by the top management.
The tree in figure 3 describes the considered problem as an extensive game. To get a solution, one has to assess the subgame perfect equilibria [20]. Two conditions must be fulfilled to get a subgame perfect equilibrium: On the one hand the strategy profile must be a Nash equilibrium for the whole game and on the other hand the actions of the players must constitute a Nash equilibrium for every subgame [18]. In every subgame any decision has to be made as if a new game is started at this node. Afterwards the rational behaviour is transferred by backward induction to the previous level [24].
To prove whether the unfavorable end node 5 can become a Nash equilibrium, the following three questions have be clarified step-by-step:

1) Under which conditions can node 2 become a subgame perfect equilibrium in the left subgame?
2) Under which conditions does node 5 dominate all the other end nodes in the right subgame regarding the payoffs?
3) Under which conditions will node 5 be a Nash equilibrium?

Hence it must be checked, under which conditions there the risk exists that an avoidable cost increase will not be detected because of bad material planning and negligent auditing by the top management.
Subsequently the payoffs $\pi_n(M)$ of the material manager and $\pi_n(C)$ of the controller at the five end nodes $n\,(n=1,...,5)$ are described:

$$\pi_1(M) = p_h \cdot p_m \cdot (Z + B_D)$$
$$\pi_1(C) = p_h \cdot p_m \cdot (V - K)$$

$$\pi_2(M) = (1 - p_h) \cdot p_m \cdot (Z + B_D)$$
$$\pi_2(C) = (1 - p_h) \cdot p_m \cdot (V)$$

$$\pi_3(M) = p_h \cdot (1 - p_m) \cdot (Z - S + L)$$
$$\pi_3(C) = p_h \cdot (1 - p_m) \cdot (V - K + B_C)$$

$$\pi_4(M) = p_a \cdot (1 - p_h) \cdot (1 - p_m) \cdot (Z - S + L)$$
$$\pi_4(C) = p_a \cdot (1 - p_h) \cdot (1 - p_m) \cdot (V - S)$$

$$\pi_5(M) = (1 - p_a) \cdot (1 - p_h) \cdot (1 - p_m) \cdot (Z + B_D + L)$$
$$\pi_5(C) = (1 - p_a) \cdot (1 - p_h) \cdot (1 - p_m) \cdot (V)$$

Step 1:
Derivation of the conditions under which node 2 becomes a subgame perfect equilibrium in the left subgame

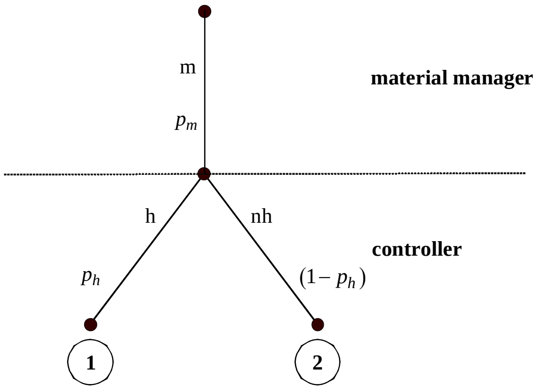

Figure 4: Subgame of the left branch

Without elaborating the estimates of all inequalities, the conditions, under which the payoffs of node 2 dominate the payoffs of node 1, will be shown below. From the manager's point of view node 2 is preferred to node 1 if and only if

$$p_h < \frac{1}{2}\ (*) .$$

The condition for the controller is fulfilled if and only if

$$p_h < \frac{V}{2 \cdot V - K}\ (**) \text{ with } K < 2 \cdot V .$$

In the whole model the costs do not only have effects on the probabilities. The inequalities do not allow simple conclusions about interactions of diversified payoffs and probabilities for getting the incident (h). Basic observations of the payoff nodes in the left subgame show that these can only be achieved by multiplication of the probabilities p_m and p_h. In the right subgame the end nodes can be determined by multiplication of the three probabilities p_m, p_h, and p_a. In the left branch the estimate for the material manager is stricter than the controller´s estimate. If (*) is fulfilled, (**) applies accordingly, because $2 \cdot V - K < 2 \cdot V$ fullfills the conditions for all positive *K* and *V*.

Step 2:
Derivation of the conditions, under which node 5 becomes a subgame perfect equilibrium in the right subgame

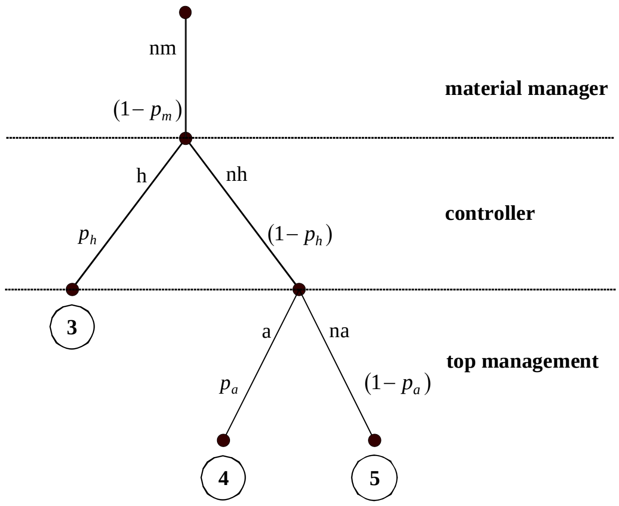

Figure 5: Subgame of the right branch

The estimates in the right branch (see figure 5) turn out to be more difficult, because three end nodes have to be compared. It must be analysed, under which conditions the material manager chooses his behaviour in the way that node 5 will become the Nash equilibrium. According to the assumption that the controller's bonus for finding the fault of the material manager is higher than the costs of a high audit the dominance of node 5 has to be analysed. If the costs of a scrutiny are higher than the additional bonus, a Nash equilibrium in pure strategies is achieved (see figure 2). When considering the payoffs of the end nodes 4 and 5, it becomes apparent that the dominance of node 5 on the one hand depends on a value basis payoff and on the other hand on the incidence of "detect the wrong behaviour of both players". Except for the probability p_a the probabilities do not differ in the left subgame. The same probabilities $(1-p_n)$ and $(1-p_h)$ exist. In the estimate the probabilities are reduced accordingly. End node 5 dominates end node 4 if and only if

$$p_a < \frac{Z + B_D + L}{\underbrace{2 \cdot Z + B_D - S + 2 \cdot L}_{(***)}}$$

It is assumed that the following conditions hold: $Z - S + L > 0$ and then $(***) > Z + B_D + L$. According to elementary economic considerations this seems to be logical, because a penalty, which is higher than the annual salary, would be irrational in games, which are not repeated.

In the next step it will be shown that a dominance of end node 5 over end node 3 holds under consideration of the prior derivated dominance statement. If and only if the following condition is fulfilled, end node 5 dominates end node 3 under condition of the defined payoffs:

$$p_a < \frac{Z + B_D + L - p_h \cdot (2 \cdot Z + B_D - S + 2 \cdot L)}{(1-p_h) \cdot (Z + B_D + L)} \; (****).$$

The following inequalities must apply to guarantee that $p_a \in [0,1]$ is fulfilled:

- $p_h < 1$,
- $p_h \cdot (Z - S + L) > 0$,
- $p_h < \dfrac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}$.

The third inequality has to be equivalent to the condition, which was derived for the probability p_a if end node 5 dominates end node 4.
After remodelling the condition (****) includes the proposition that the probability p_a of decreases with an increasing probability p_h of a high auditing level. So, the following condition has to be fulfilled: ${}^{dp_a}\!/\!{}_{dp_h} < 0$ (see figure 6).

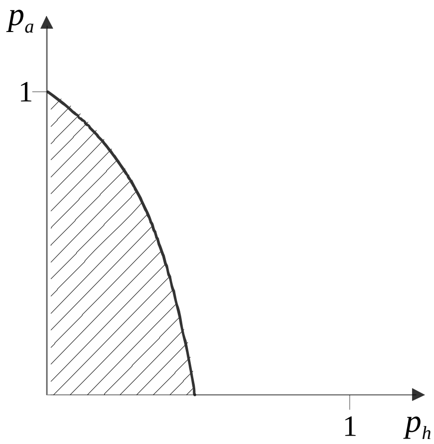

Figure 6: Values of the probabilities for the subgame in the right branch from the manager's view

Likewise the estimates of the payoffs for getting a Nash equilibrium in end node 5 have to be analysed from the controller's point of view. One has to analyse under which conditions node 5 dominates nodes 3 and 4. This holds if and only if the following conditions are fulfilled.
End node 5 dominates 4:

- $p_a < \dfrac{V}{2 \cdot V - S}$ (1).

End node 5 dominates 3:

- $p_a < \dfrac{V - p_h \cdot (2 \cdot V - K + B_C)}{(1 - p_h) \cdot V}$ (2),
- $p_h < 1$,
- $p_h \cdot (V - K + B_C) > 0$,
- $p_h < \dfrac{V}{2 \cdot V - K + B_C}$ (3).

It is also obvious, that the probability p_a, to detect the wrong behaviour of the manager and the controller, decreases with a higher probability p_h. The gradient has the characteristics

${dp_a}/{dp_h} < 0$, too. The scopes of the probabilities p_a and p_h which fulfil the above conditions (1) – (3) are sketched in figure 7.

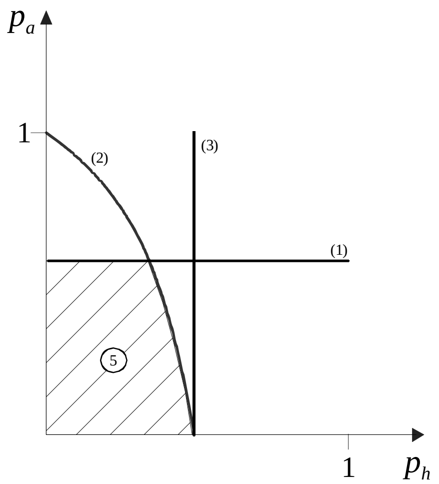

Figure 7: Scopes of the probabilities p_a and p_h for end node 5 as the subgame perfect equilibrium in the right branch

Before discussing the conditions under which end nodes 2 or 5 becomes a Nash equilibrium, it must be analysed first how the probabilities p_a and p_h have to be distributed to realise node 3 or 4 as a subgame perfect equilibrium. The relevant values are given by table 1 below. A graphical visualisation is shown by figure 8. For combinations of the probabilities p_a and p_h the Nash equilibria of the end nodes 3, 4 and 5 will be shown. These are described by the white areas ③, ④ and ⑤ respectively. It can easily be seen that the probability p_a will take a value less than zero if the probability p_h tends towards the value 1. This consideration can be explained by the exogenously defined condition that the probability p_a only exists in the end nodes 4 and 5. In the other nodes this probability takes the value zero or one. Now it is elementary to find out, under which conditions end node 3 becomes a Nash equilibrium in this game. On the basis of the results it can easily be verified to which extent the inequalities of the controller are stricter than the ones of the material manager in the right subgame. If and only if Z takes a high value, these conclusions can be partly violated.

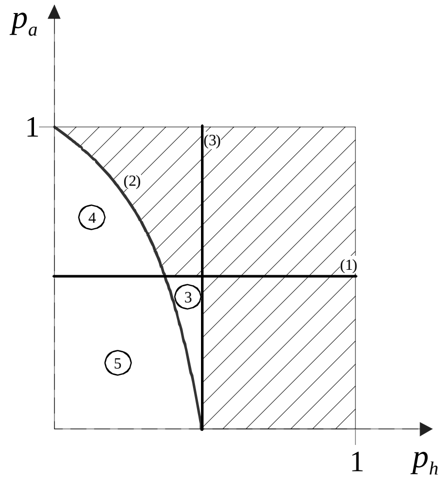

Figure 8: Distribution of the probabilitites p_a and p_h for a subgame perfect equilibrium in the right branch

Table 1: Conditions for which end nodes 3 to 5 become a subgame perfect equilibrium

Conditions for end node 3:

Material manager

$$p_a < \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}, \qquad \pi_5(M) > \pi_4(M),$$

$$p_a > \frac{Z + B_D + L - p_h \cdot (2 \cdot Z + B_D - S + 2 \cdot L)}{(1 - p_h) \cdot (Z + B_D + L)}, \qquad \pi_3(M) > \pi_5(M),$$

$$p_h < \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}.$$

Controller

$$p_a < \frac{V}{2 \cdot V - S}, \qquad \pi_5(C) > \pi_4(C),$$

$$p_a > \frac{V - p_h \cdot (2 \cdot V - K + B_C)}{(1 - p_h) \cdot V}, \qquad \pi_3(C) > \pi_5(C),$$

$$p_h < \frac{V}{2 \cdot V - K + B_C}.$$

Conditions for end node 4:

Material manager

$$p_a > \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}, \qquad \pi_4(M) > \pi_5(M),$$

$$p_a < \frac{Z + B_D + L - p_h \cdot (2 \cdot Z + B_D - S + 2 \cdot L)}{(1 - p_h) \cdot (Z + B_D + L)}, \qquad \pi_5(M) > \pi_3(M),$$

$$p_h < \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}.$$

Controller

$$p_a > \frac{V}{2 \cdot V - S}, \qquad \pi_4(C) > \pi_5(C),$$

$$p_a < \frac{V - p_h \cdot (2 \cdot V - K + B_L)}{(1 - p_h) \cdot V}, \qquad \pi_5(C) > \pi_3(C),$$

$$p_h < \frac{V}{2 \cdot V - K + B_C}.$$

Conditions for end node 5:

Material manager

$$p_a < \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}, \qquad \pi_5(M) > \pi_4(M),$$

$$p_a < \frac{Z + B_D + L - p_h \cdot (2 \cdot Z + B_D - S + 2 \cdot L)}{(1 - p_h) \cdot (Z + B_D + L)}, \qquad \pi_5(M) > \pi_3(M),$$

$$p_h < \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}.$$

Controller

$$p_a < \frac{V}{2 \cdot V - S}, \qquad \pi_5(C) > \pi_4(C),$$

$$p_a < \frac{V - p_h(2 \cdot V - K + B_C)}{(1 - p_h) \cdot V}, \qquad \pi_5(C) > \pi_3(C),$$

$$p_h < \frac{V}{2 \cdot V - K + B_C}.$$

Step 3:
Derivation of the conditions, under which node 5 becomes a Nash equilibrium in the game

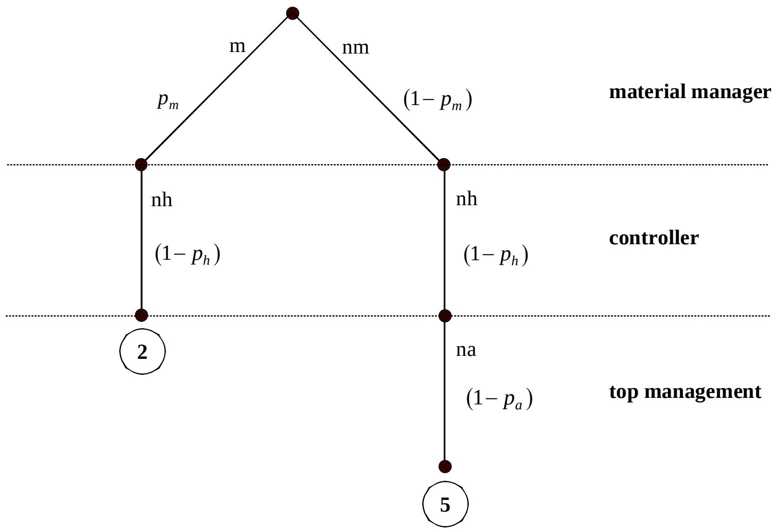

Figure 9: Comparison of the left and right branch

After step 1 and 2 it must be analysed, under which conditions end node 5 dominates end node 2. This applies if and only if

- $p_a < \dfrac{Z + B_D + L - p_m \cdot (2 \cdot Z + 2 \cdot B_D + L)}{(1 - p_m) \cdot (Z + B_D + L)} \Rightarrow p_m < \dfrac{Z + B_D + L}{2 \cdot Z + 2 \cdot B_D + L} < 1$ and

  $Z + B_D > 0$

  (material manager),

- $p_a < \dfrac{1 - 2p_m}{1 - p_m} \Rightarrow p_m < 0,5$

  (controller).

If Z, B_D and L are positive, the estimates of the controller dominate the manager's ones. The correlations between p_a and p_m (see figure 10) are structured analogously to the function delineated in figure 6. There exists no relation between the probability p_m and the probability p_h in node 5, because the controller shows the same behaviour in the end nodes 2 and 5. Conclusions about implications between p_m and p_h can be derived comparing end node 1 to end node 5 or end node 2 to end node 3. It can be also concluded that a comparison of end node 1 to end node 5 directly induces that the probability of a high audit level decreases with an increasing probability of a methodical decision making of the material manager. The estimation between the end nodes 2 and 3 also shows that the material manager will work methodically correct if the controller audits the data more intensively: With an increasing probability p_h the probability p_m increases, too.

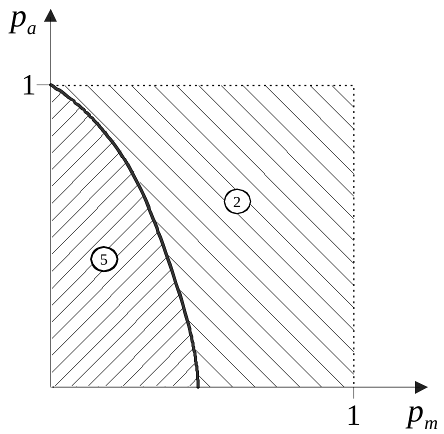

Figure 10: Conditions for end node 5 to become a Nash equilibrium in the total model

Considering the areas ② and ⑤ in figure 10 it becomes apparent, for which probabilities p_a and p_m the non-methodical decision of the material manager can turn into a Nash equilibrium of the total model under a low auditing level. The consequence will be a loss for the company pictured by area ⑤. Combinations of the probabilities p_a and p_m in area ② lead to a Nash equilibrium of end node 2 (see figure 10).
Closing these considerations it will be shown that the constellation of the probabilities and payoffs has to be determined endogenously so that end node 1 becomes a Nash equilibrium and the two considered players behave correctly and the company maximizes its profit.
Step 1 is the basis for end node 1 becoming a subgame perfect equilibrium if and only if

for the material manager: $p_h > \frac{1}{2}$,

for the controller: $p_h > \frac{V}{2 \cdot V - K}$.

In step 2 it was shown, that end node 3 becomes a subgame perfect equilibrium if and only if

for the material manager:

$$p_a < \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L},$$

$$p_a > \frac{Z + B_D + L - p_h \cdot (2 \cdot Z + B_D - S + 2 \cdot L)}{(1 - p_h) \cdot (Z + B_D + L)},$$

$$p_h < \frac{Z + B_D + L}{2 \cdot Z + B_D - S + 2 \cdot L}.$$

for the controller:

$$p_a < \frac{V}{2 \cdot V - S},$$

$$p_a > \frac{V - p_h \cdot (2 \cdot V - K + B_C)}{(1 - p_h) \cdot V},$$

$$p_h < \frac{V}{2 \cdot V - K + B_C}.$$

A comparison of the end nodes 1 and 3 results in the conclusion that end node 1 becomes a Nash equilibrium of the modified “inspection game” if and only if

for the material manager: $$p_m > \frac{Z - S + L}{2 \cdot Z + B_D - S + L},$$

for the controller: $$p_m > \frac{V - K + B_C}{2 \cdot V - 2 \cdot K + B_C}.$$

5 Summary

If the characteristic traits of all players are unknown, it is of interest for the company to make an analysis of deviation in different divisions like procurement. The result of these audits will be that correct work was done or that the company has to learn its lesson from not achieving its goals. Hence, an analysis of deviation makes sense in order to detect incorrect behaviour of the companies’ employees ex-post. However, if the top management does not work intensively and does not monitor the work on a high level, the incorrect work of the material manager and the controller will not be detected and the profit will decrease because of high costs by the procurement. A fundamental job of the top management is to avoid this situation. By choosing the appropriate level of the parameters – bonus and punishment – the top management can stimulate the two players to perform on a high level and to maximize the company’s profit (reaching end node 1). Anyhow, it was shown that a Nash equilibrium can be generated, which determines the probabilities of the two players meeting in end node 5 because of a wrong stock level endogenously.

References

1. Avenhaus, R., Cauty, M., Kilgour, D. M., von Stengel, B., Zamir, S., 1996. Inspection Games in Arms Control. *European Journal of Operations Research*, 90, 383-394.
2. Avenhaus, R., Stengel, B., Zamir, S., 2002. Inspection Games. In: Aumann, R. J., Hart, S., (Eds.). *Handbook of Game Theory*, 3, Amsterdam: North-Holland, 1947-1987.
3. Avenhaus, R., 2002. Game Theory. In: Kremenyuk, V., (Ed.). *International Negotiation*. San Fransisco: Jossey Bass 2nd Edition, 202-228.
4. Biermann, B., 2006. *Die Anwendung spieltheoretischer Methoden zur Definition eines optimalen Kontrolldesigns*. Dissertation, Norderstedt: Books on Demand GmbH.
5. Borch, K., 1982. Insuring and Auditing the Auditor. In: Deistler, M., Fürst, E., Schwödiauer, G., (Eds.). *Games, economic dynamics, time series analysis*. Wien: Physica, 117-126.
6. Dresher, M., 1962. *A sampling inspection problem in arms control agreements: a game-theoretic analysis*. Memorandum No. RM-2972-ARPA, Washington: RAND Corporation.
7. Ewert, R., 1992. Controlling, Interessenkonflikte und asymmetrische Information. *Betriebswirtschaftliche Forschung und Praxis*, 44, 277-303.
8. Ewert, R., 1993. Rechnungslegung, Wirtschaftsprüfung, rationale Akteure und Märkte: ein Grundmodell zur Analyse der Qualität von Unternehmenspublikationen. *Zeitschrift für betriebswirtschaftliche Forschung*, 45, 715-747.
9. Ewert, R., Stefani, U., 2001. Wirtschaftsprüfung. In: Jost, P. J., (Ed.). *Die Spieltheorie in der Betriebswirtschaftslehre*. Stuttgart: Schäffer-Poeschel, 175-217.
10. Fandel, G., Trockel, J., 2007. Bilanzpolitik und Wirtschaftsprüfung im Licht neuerer spieltheoretischer Ansätze. In: Winkeljohann, N., Bareis, P., Volk, G., (Eds.). *Rechnungslegung, Eigenkapital und Besteuerung – Entwicklungstendenzen. Festschrift für Dieter Schneeloch zum 65. Geburtstag*. München: Vahlen, 167-184.
11. Jost, P.-J., 1999. *Strategisches Konfliktmanagement in Organisationen*. Wiesbaden: Gabler.
12. Harris, F. W., 1913. How many parts to make at once. *Factory, The Magazine of Management*, 10, 135-136 and 152. Reprinted in: *Operations Research*, 38, 1990, 947-950.
13. Kuhn, H., 1963. Recursive Inspection Games. In: Anscombe, F. J., (Ed.). *Applications of Statistical Methodology to Arms Control and Disarmament*. Final report to the U.S. Arms Control and Disarmament Agency under contract No. ACDA/ST-3 by Mathematica, Inc., Princeton, New Jersey, Part III, 169-181.
14. Maschler, M., 1966. A price leadership method for solving the inspectors non-constant-sum game. *Naval research logistics quarterly*, 13, 11-33.
15. Maschler, M., 1967. The inspector´s non-constant-sum game: Its dependence on a system of detectors. *Naval research logistics quarterly*, 14, 275-290.
16. Nash, J., 1950. The Bargaining Problem. *Econometrica*, 18, 155-162.

17. Nash, J., 1951. Non-cooperative Games. *Annals of Mathematics*, 54, 286-295.

18. Riechmann, T., 2002. *Spieltheorie.* München: Vahlen.

19. Rinderle, K., 1996. *Mehrstufige sequentielle Inspektionsspiele mit statistischen Fehlern erster und zweiter Art*. Dissertation, Hamburg: Kovač.

20. Selten, R., 1975. Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games. *International Journal of Game Theory*, 4, 25-55.

21. Stadtler, H., 2007. How important is it to get the lot size right?. *Zeitschrift für Betriebswirtschaft*, 77, 407-416.

22. Tempelmeier, H., 2006. *Material-Logistik*. Berlin: Springer.

23. von Stengel, B., 1991. *Recursive Inspection Games*, Technical Report S-9106, Universität der Bundeswehr München.

24. Wiese, H., 2002. *Entscheidungs- und Spieltheorie*. Berlin: Springer.