Green energy trade and unilateral climate policy*

Thomas Eichner
Department of Economics, University of Hagen

Gilbert Kollenbach
School of Economics, University of East Anglia

Abstract

In a dynamic Hotelling model with two (groups) of countries, fossil fuel, green energy trade and unilateral climate policy, the dynamics of the economy are analyzed. Countries are different with respect to their climate policy and green energy production costs, and energy trade is associated with conversion losses. In a calibrated economy, green energy trade may end before the fuel stock is exhausted. Comparing the fuel extraction paths with green trade and under green autarky shows that green energy trade flattens the fuel extraction path if conversion losses are negligible. The fuel extraction path is characterized by an intratemporal paradox of green trade at later periods and an intertemporal orthodox at earlier periods. If conversion losses are significant, energy trade may steepen the fuel extraction path and an intratemporal paradox at early periods ensues.

JEL classification: H23, Q54, Q58
Key words: trade, fossil fuel, green energy, unilateral policy

*Eichner: Department of Economics, University of Hagen, Universitaetsstr. 41, 58097 Hagen, Germany, email: thomas.eichner@fernuni-hagen.de; Kollenbach: School of Economics, University of East Anglia, Norwich Research Park, Norwich NR4 7TJ, UK email: g.kollenbbach@uea.ac.uk.

1 Introduction

The European Union has committed to reduce its greenhouse gas emissions by 55 % by 2030 compared to 1990 and to be climate-neutral by 2050 (European Parliament 2022).[1] The European Commission's Renewable Energy Directive requires the EU to cover 32% of its energy consumption by renewable energy. In 2020, about 17% of the gross available energy in the EU was generated by domestic renewable energy. In addition, the EU is an energy importer. In 2020, the EU has covered 58% of its energy demand by energy imports, mainly fossil fuel energy.[2] To achieve its ambitious goals, the European Commission argues that imports of green energy have to be massively expanded. The European Parliament (2022) writes: "A broad-based strategy for the importation of renewable electricity, renewable hydrogen and low-carbon energy from as many naturally suitable regions as possible is necessary, also to reduce fossil dependencies."

The present paper is the first that analyzes unilateral climate policy in a dynamic multi-country Hotelling model with fossil fuel and green energy trade. In that model, homogeneous fossil fuels are extracted at constant marginal costs and renewable (green) energy is a perfect substitute in energy consumption. There are two groups of countries. The climate country levies a fuel tax,[3] whereas the other country abstains from any climate policy. Furthermore, countries differ in their green energy production costs. Both fossil fuels and green energy are internationally mobile, but the trade of green energy is associated with transport costs (Collis and Schomaecker 2022) and conversion losses (Schrotenboer et al. 2022). Within different scenarios we study when green energy is traded and how the fuel tax and green trade influence the fuel extraction path.

There is large literature that studies the socially optimal extraction path of fossil fuels and the first-best climate policies such as carbon or fossil fuel taxes in dynamic Hotelling models. Hoel and Kverndokk (1996) and Tahvonen (1997) investigate the extraction of fossil fuels in the presence of a clean backstop when the fossil fuel stock causes a pollution externality in an one-country model. Costs of renewables are linear and extraction costs are stock dependent. They show that in the social optimum there exists a phase in which both fossil fuel and the backstop are used, whereby fossil fuel consumption decreases and the backstop consumption increases in time. Also in a dynamic one-country Hotelling model Chakravorty et al. (2006) characterize the optimal path of fossil fuel extraction and backstop

[1]Not only the European Union but als China and the United States of America plan to be climate-neutral by the middle of this century

[2]All data are from Eurostat.

[3]In our model the fuel tax is equivalent to a carbon emissions tax.

generation when there is a ceiling on the stock of pollution.

Turning to two-country models, Hoel (2011), Ryszka and Withagen (2016) and Kollenbach (2019) investigate the effect of exogenous changes in the climate policy on the fuel extraction path. The effects of exogenous changes are related to the notion of the green paradox. If a tightening of the climate policy today increases fuel extraction today a weak green paradox arises. If a tightening of the climate policy today increases the present value of the intertemporal climate damage, a strong green paradox arises. The above mentioned articles consider both fossil fuels and renewable energy whereby the backstop costs are linear. The articles differ with respect to the assumptions regarding the extraction costs. Hoel (2011) abstracts from extraction costs. In Ryszka and Withagen (2016) extraction costs are linear in extracted fuel and in Kollenbach (2019) extraction costs are convex and flow- and stock-dependent. The answer whether or not a green paradox emerges depends on whether the climate policy of the high-tax or low-tax country is tightened, on the size of the fuel stock and on the price elasticity of energy demand.

Van der Ploeg (2016) and Kollenbach and Schopf (2022) go one step further and characterize the unilaterally optimal policy in Hotelling models. Van der Ploeg (2016) applies a two-period two-country Hotelling model and Kollenbach and Schopf (2022) use a dynamic two-country Hotelling model. Some important results in these articles are that uniltaterally optimal carbon taxes of fuel-importing countries exceed first-best carbon taxes and that both a weak and a strong green paradox can be associated with the unilaterally optimal climate policies. Eichner et al. (2023) compare unilaterally optimal demand- and supply-side climate policies to adhere a carbon ceiling. In the literature discussed so far markets are perfectly competitive. The impact of monopolistic supply of exhaustible resources and substitutes on climate policy is analyzed by Andrade de Sá and Daubanes (2016), van der Meijden et al. (2018), van der Meijden and Withagen (2019) and Curuk and Sen (2023). None of the previously mentioned papers analyzes green energy trade between countries.

In the present paper, we first investigate a laissez-faire economy, i.e. an economy without any regulation, in which countries differ with respect to their backstop production costs. Initially, both countries use both fossil energy and green energy. When fossil fuel becomes scarcer, fossil fuel extraction expires in the country with the lower backstop costs. The fuel scarcity increases in time with the consequence that the country with the lower backstop costs exports green energy. In the last phase of the economy, fossil fuel expires also in the country with the larger backstop costs and energy consumption is covered completely by green energy. Comparing the laissez-faire economy with green trade with a laissez-faire economy under green autarky, green trade steepens the fuel extraction path such that more fuel is instantaneously used and instantaneous carbon emissions increase. We call this effect *intertemporal paradox of green trade* at early periods. The laissez-faire economy is inefficient due to a climate externality. However, comparing the laissez-faire economy with the social optimum shows that both have the same dynamics of of fuel extraction and the same pattern of green energy flows.

Next, we study an economy in which one country sets a unilateral fuel tax and behaves non-strategically, i.e. ignores the impact of its climate policy on the market prices and the market allocation. We identify two timings that are especially relevant for our empirical calibration. We denote the country with the unilateral emissions tax and the larger backstop costs by country A and the country with no emissions regulation and the lower backstop costs by country B. At the first timing, initially country B exports its total produced green energy to country A and initially both countries use fossil fuels but only country A consumes green energy. With an increasing fuel scarcity rent, in country A green energy crowds out fossil fuel consumption and country A opts out fuel use. When time progresses country B divides its green energy production between exports and domestic green energy consumption and later on the fuel stock is exhausted. In the steady-state both countries produce green energy and country B exports some of its green energy to country A. Compared to a green autarky economy with a unilateral fuel tax, green trade reduces fossil fuel extraction in early periods and raises fuel extraction in later periods, i.e. there is an intertemporal orthodox of green trade at early periods. The effect that green trade increases fuel extraction at later periods is called *intratemporal paradox of green trade* at later periods.

At the second timing, green energy trade ends before the first of the two countries, which is country B, opts out fossil fuel use. More specifically, initially country B exports all of its green energy to country A. As time passes, country B splits its green energy production to exports and domestic green energy consumption with a decreasing share of exports such that green energy exports run out. After green energy trade has ended, country B stops fuel consumption before the fuel stock is exhausted and fuel consumption vanishes in country A. Compared to the green autarky economy with unilateral fuel tax, an intratemporal paradox of green trade at mean periods and an intertemporal orthodox of green trade either at early periods or at late periods arises.

So far, we have assumed that the country that sets the fuel tax behaves as price-taker in the markets. However, in case the country acts strategically, i.e. manipulates the market prices and the market allocation in its favor, we identify three strategic effects: (i) the terms-of-trade effect with respect to fuel, (ii) the terms-of-trade effect with respect to green energy and (iii) the emissions effect. At the emissions effect (iii), the taxing country has an incentive to reduce the fuel tax in order to increase the energy price and to mitigate carbon leakage to the other country. In the empirical calibration the terms-of-trade effects dominate the emissions effect.

Finally, we calibrate the model to the world oil market and consider e-fuels as green energy substitute. We consider two scenarios. One realistic scenario at which the conversion of e-fuels is not associated with any losses and one hypothetical scenario in which the conversion of e-fuels requires a conversion loss in the amount of 20%.[4] In all scenarios fossil fuel extraction decreases over time and total green energy production increases over time. More specifically, in the realistic scenario the first timing holds both for non-strategic and strategic behavior. The main finding is that green trade flattens the extraction path and there is an intertemporal orthodox at early periods and an intratemporal paradox at late periods. The hypothetical scenario is characterized by the second timing if country A behaves non-strategically. It turns out that the extraction path is steeper with trade than under autarky. An intratemporal paradox at early and at mean periods arises. Strategic effects are so strong that the results concerning the extraction path are reversed, i.e. strategic effects flatten the fuel extraction path with green trade such that the fuel extraction path with green trade is flatter than the fuel extraction path under green autarky.

The remainder of the paper is organized as follows. Section 2 outlines the model. Section 3 characterizes the competitive economy in the absence of any regulation (laissez-faire economy) and the social optimum and compares both. In section 4 we analyze the unilaterally fuel tax with and without strategic action and provide some general results for two specific timings. Section 5 contains an empirical calibration. Section 6 concludes.

2 The model

Consider an economy with two (groups of) countries A and B. At time t the representative consumer of country $i = A, B$ derives utility from consuming the amount $y_i(t)$ of energy and the amount $x_i(t)$ of a composite consumer good. Her preferences are represented by the utility function

$$U(y_i(t)) + x_i(t) = \frac{a}{z} y_i(t) - \frac{y_i(t)^2}{2z} + x_i(t), \tag{1}$$

[4]Although the conversion loss does not fit to e-fuels there are other green energies such as green hydrogen or green ammonia which have significant conversion losses.

with the parameters $a, z > 0$. Energy $y_i(t)$ composes of energy from fossil fuels (black energy) $b_i^d(t)$ and energy from renewables (green energy) $g_i^d(t)$. Black and green energy are perfect substitutes such that at every point in time t energy demand of country i is

$$y_i(t) = b_i^d(t) + g_i^d(t). \tag{2}$$

The fuel endowment of the economy is given by S_0. The share s_A of this endowment is located in country A and the remaining share $s_B = 1 - s_A$ in country B. Fuel extraction is proportional to black energy generation and we denote by $b_i^s(t)$ the black energy generation of country i. The extraction and production costs are

$$C(b_A^s(t) + b_B^s(t)) = c\left[b_A^s(t) + b_B^s(t)\right], \tag{3}$$

where $c > 0$ are the constant marginal costs. The evolution of the fossil fuel stock over time is given by

$$\dot{S} = -b_A^s - b_B^s. \tag{4}$$

Black energy consumption generates carbon emissions. We denote by $b_i^d(t)$ both black energy consumption and carbon emissions of country i at period t, and by $E(t)$ the atmospheric CO_2 stock at period t. The motion of the carbon emissions stock is given by

$$\dot{E} = b_A^d + b_B^d - \gamma E. \tag{5}$$

It increases in total black energy consumption, decays at a constant rate γ, and causes the climate damage

$$H(E(t)) = hE(t), \tag{6}$$

where $h > 0$ is the constant marginal damage.

Next to black energy, the countries produce green energy. We assume that the production locations of green energy differ with respect to their costs and that country i's costs of green energy generation are given by

$$M_i(g_i^s(t)) = \frac{m_i}{2}\left[g_i^s(t)\right]^2, \tag{7}$$

where $m_i > 0$ is a country-specific cost parameter. W.l.o.g. we assume $m_A \geq m_B$. The produced green energy $g_i^s(t)$ in country $i = A, B$ can be either used domestically or exported to country $j \neq i$. g_{ii}^s units of the green energy production g_i^s are consumed in country i and g_{ij}^s units are exported to country j, so that $g_i^s = g_{iA}^s + g_{iB}^s$. To transport green energy over long distances it has to be converted which is both costly and associated with conversion losses. The transport costs of exporting green energy from country i to country j are given by

$$Q(g_{ij}^s(t)) = \frac{q}{2}\left[g_{ij}^s(t)\right]^2, \tag{8}$$

where q is a cost parameter. Furthermore, the fraction $(1-\alpha)$ of each unit exported green energy is lost by conversion such that the remaining fraction α of each green energy unit is available for consumption in the country of destination.

The description of the model is completed by the black and green energy constraints

$$\begin{aligned} b_A^s(t) + b_B^s(t) &= b_A^d(t) + b_B^d(t), && (9) \\ g_{ii}^s(t) + \alpha g_{ji}^s(t) &= g_i^d(t) \qquad \forall\, i = A, B,\, i \neq j, && (10) \end{aligned}$$

and the constraint for the composite consumer good

$$x_A(t) + x_B(t) = \sum_i \left[\bar{\ell}_i - C(b_i^s(t)) - Q(g_{ij}^s(t)) - M_i(g_i^s(t))\right]. \tag{11}$$

In (11), $\bar{\ell}_i$ is country i's endowment of a domestic resource as land or labor that is input in the production of the consumer good and used for the generation and transport of energy.[5] The term $\bar{\ell}_i - C(b_i^s(t)) - Q(g_{ij}^s(t)) - M_i(g_i^s(t))$ is country i's possibility frontier of transforming energy into the consumer good. The consumer good and black energy are internationally traded. Green energy is also internationally traded, but the mobility of green energy causes transport costs and conversion losses.

3 Laissez-faire economy and social optimum

In subsection 3.1 we investigate the laissez-faire economy, in which climate policies are absent, before we turn in subsection 3.2 to the social optimum, leading up to a brief comparison between the laissez-faire economy and the social optimum.

3.1 Laissez-faire economy

In the economy there is an international and perfectly competitive market for energy. Fossil fuel is supplied by a representative fuel firm. In each country resides a representative household and operates a representative green energy firm. Because the transport of green

[5]For more details we refer to the microfoundation of our model presented in the Appendix A.1.

energy is associated with costs and conversion losses, the energy markets in country A and B are segmented and we introduce the consumer energy prices $(p_A(t), p_B(t))$ and the producer prices $(p^b(t), p_A^g(t), p_B^g(t))$, where $p^b(t)$ is the black energy producer price and $p_i^g(t)$ is the green energy producer price in country $i = A, B$.

Denoting ρ the time-preference rate, the fossil fuel firm maximizes its intertemporal profits $\int_0^\infty e^{-\rho t}\Pi_F(t)\mathrm{d}t$, where

$$\Pi_F(t) = p^b(t)\left[b_A^s(t) + b_B^s(t)\right] - c\left[b_A^s(t) + b_B^s(t)\right] \tag{12}$$

subject to (4), $\int_0^\infty b^s(t)dt \leq S_0$ and the non-negativity conditions $b_A^s(t) \geq 0$, $b_B^s(t) \geq 0$ and $S(t) - b_A^s(t) - b_B^s(t) \geq 0$. Solving the corresponding Lagrangian, the fuel firm's black energy supply at time t is given by the correspondence

$$b^s(t) = \begin{cases} 0, & \text{if } p^b(t) < c + \lambda(t), \\ b_A^s(t) + b_B^s(t) \in [0, S(t)], & \text{if } p^b(t) = c + \lambda(t), \\ b_A^s(t) + b_B^s(t) = S(t), & \text{if } p^b(t) > c + \lambda(t), \end{cases} \tag{13}$$

where λ denotes the scarcity rent (costate of the fuel stock). The black energy supply is driven by the linearity of the extraction costs. If the fuel price is below [above] the sum of marginal extraction cost c and scarcity rent $\lambda(t)$ the fuel firm does not supply any energy [supplies the whole fossil fuel stock] at time t. In case of an interior solution, the fuel firm is indifferent between selling any amount of black energy.[6] According to the Hotelling-rule

$$\hat{\lambda} = \rho \tag{14}$$

the scarcity rent increases with the time preference rate ρ. The transversality condition

$$\lim_{t\to\infty} e^{-\rho}\lambda(t)\left[S(t) - S^{opt}(t)\right] \geq 0 \tag{15}$$

ensures that the complete fuel stock is extracted.[7]

The representative green energy firm of country $i = A, B$ maximizes its profits

$$\Pi_i(t) = p_i^g(t)g_{ii}^s(t) + p_j^g(t)\alpha g_{ij}^s(t) - M_i\left(g_{ii}^s(t) + g_{ij}^s(t)\right) - Q\left(g_{ij}^s(t)\right) \tag{16}$$

at every point in time, subject to the non-negativity conditions $g_{ii}^s(t) \geq 0$ and $g_{ij}^s(t) \geq 0$. In (16), $p_i^g(t)g_{ii}^s(t)$ are the revenues from selling green energy in country i and $p_j^g(t)\alpha g_{ij}^s(t)$

[6] In that case, the equilibrium fossil fuel extraction is purely determined by the demand side.

[7] The superscript ^{opt} marks the optimal evolution path, while the unmarked path refers to every feasible path.

are the revenues from selling green energy in country $j \neq i$. $M_i\left(g_{ii}^s(t)+g_{ij}^s(t)\right)$ are the production costs and $Q\left(g_{ij}^s(t)\right)$ are the transport costs from delivering green energy from country i to country j. In an interior solution the first-order conditions are

$$p_i^g(t) = M_i'\left(g_{ii}^s(t)+g_{ij}^s(t)\right), \tag{17}$$

$$\alpha p_j^g(t) = M_i'\left(g_{ii}^s(t)+g_{ij}^s(t)\right)+Q'\left(g_{ij}^s(t)\right). \tag{18}$$

(17) and (18) determine firm i's green energy supply on the energy markets of countries i and j. Firm i chooses its supply such that prices are equal to marginal costs. On the domestic market, the marginal costs are the marginal production costs M_i'. On the foreign market, the marginal costs are the sum of marginal extraction costs and marginal transport costs $M_i'+Q'$. These are related to the production units α which are not lost by conversion. Because of the transport costs and the conversion losses, the firm located in country i is only willing to export green energy, if the green energy producer price in country j exceeds the green energy producer price in country i.

The representative consumer of country $i = A, B$ maximizes her utility $U\left(b_i^d(t)+g_i^d(t)\right)$ $+x_i(t)$ subject to the budget constraint $p_i(t)\left[b_i^d(t)+g_i^d(t)\right]+x_i(t)=\omega_i$, where we have normalized the price of the consumption good to unity and where

$$p_i(t)=\min\left\{p^b(t), p_i^g(t), \frac{p_j^g(t)}{\alpha}\right\}. \tag{19}$$

Because green and black energy are perfect substitutes and consumers have no green preferences, they always purchase the cheapest energy. ω_i is the consumer's income. It is given by $\omega_i=\bar{\ell}_i+\Pi_i(t)+s_i\Pi_F(t)$ and consists of an exogenous resource income $\bar{\ell}_i$, the profit $\Pi_i(t)$ of country i's green energy firm and the share s_i of the fuel firm's profit $\Pi_F(t)$. The first-order condition of utility maximization

$$U'\left(b_i^d(t)+g_i^d(t)\right)=p_i(t) \tag{20}$$

determines country i's demand for black and green energy[8] $D(p_i(t))=U'^{-1}(p_i(t))$. Eq. (20) requires at the margin that the benefit of consuming energy in country i equals the consumer energy price $p_i(t)$ from (19).

In the laissez-faire economy the dynamics of fossil fuel extraction and the pattern of green trade divide the time line into different phases which are depicted in Fig. 1. In that figure T_i is the moment in time fossil fuel consumption ends in country $i = A, B$ and t_a is a point in time that will be specified later.

[8] U'^{-1} is the inverse of the marginal utility function U'.

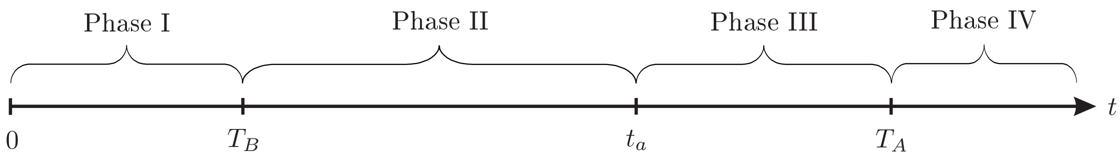


Figure 1: Timeline and sequence of phases

In the laissez-faire economy, the initial scarcity rent is λ_0, the scarcity rent path is given by $\lambda(t) = \lambda_0 e^{\rho t}$, and the corresponding fuel extraction path $b^s(t)$ meets $\int_0^T b^s(t)dt = S_0$, with $T = \max\{T_A, T_B\}$ as the point in time fossil fuel extraction vanishes. To characterize the phases of the laissez-faire economy we assume that the marginal extraction costs c are sufficiently low and the fuel endowment is sufficiently large. As a consequence, the fuel firm can undercut the green energy firms' supply prices that would prevail in a laissez-faire market equilibrium with green energy supply only and in Phase I both black energy and green energy are supplied in country A and in country B at the producer energy price[9] $p^b = p^g_A = p^g_B$. In Appendix A.2 we prove

Lemma 1 *In the laissez-faire economy, both energy sources are used in both countries and countries do not trade green energy for* $t \in [0, T_B)$.

The market equilibrium characterized by Lemma 1 is illustrated in Fig. 2. Until time T_B, both energy sources are used in both countries, so that the equilibrium energy price[10] $p^{\text{LF}}_A = p^{\text{LF}}_B$ is determined by the marginal fuel costs consisting of the marginal extraction costs c and the scarcity rent λ. Equilibrium energy use in country i is determined by the intersection of the $(c+\lambda)$-line with the demand curve $D(p_i)$, which yields total energy $b^{\text{LF}}_i + g^{\text{LF}}_{ii}$ in country $i = A, B$. The intersection of the $(c+\lambda)$-line and the marginal green energy cost curve M'_i yields country i's production and consumption of green energy.[11] The difference between total energy $b^{\text{LF}}_i + g^{\text{LF}}_{ii}$ and green energy ${g_{ii}}^{\text{LF}}$ is covered by black energy b^{LF}_i. As long as the energy prices in country A and B are identical, transport costs and conversion losses of energy exports cannot be recouped and green energy is not traded between countries.

Due to the Hotelling-rule (14), the scarcity rent increases in time and shifts upwards the $(c+\lambda)$-line. In view of Fig. 2, fuel [green energy] use decreases [increases] in both

[9]Prices and quantities depend on the time t. In the following, we omit the variable t whenever there is no risk of confusion.

[10]The allocation and prices of the laissez-faire market equilibrium are marked by the superscript LF.

[11]Because the marginal green energy costs fall short of $c+\lambda$ for $g^s_{AA} \le g^{\text{LF}}_{AA} = M'^{-1}_A(c+\lambda)$ and $g^s_{BB} \le g^{\text{LF}}_{BB} = M'^{-1}_B(c+\lambda)$, g^{LF}_{AA} and g^{LF}_{BB} are green energy use in country A and B, respectively, and green energy trade does not take place.

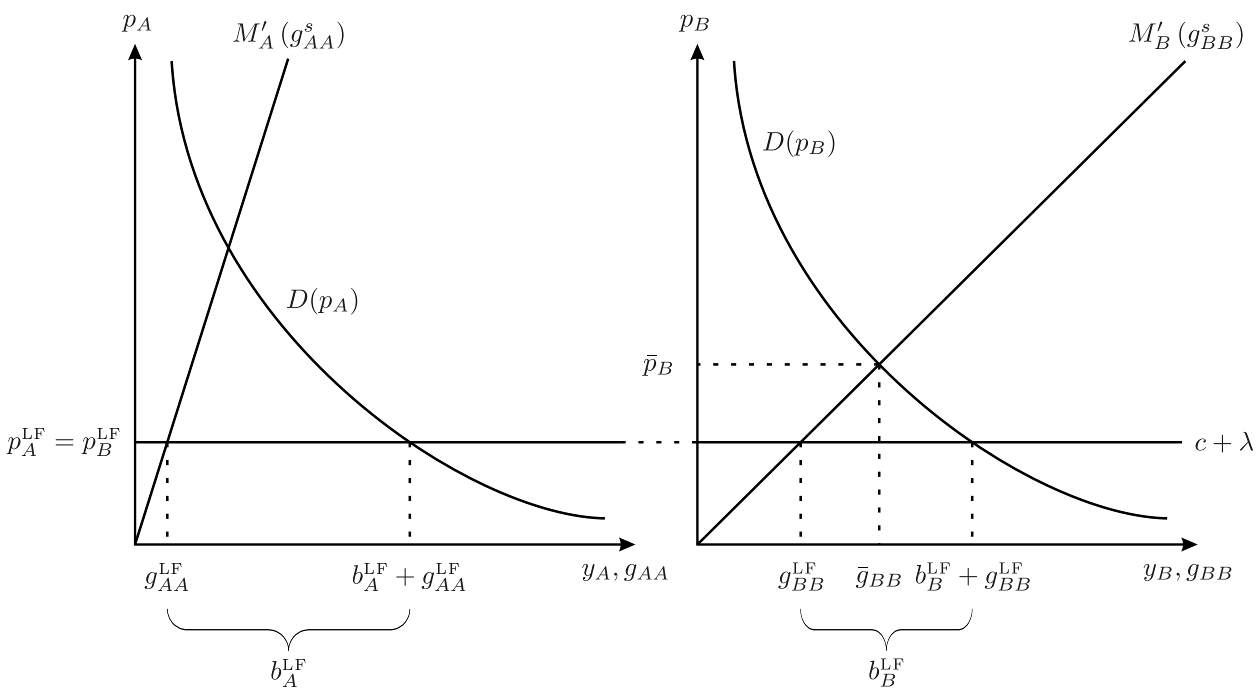


Figure 2: Energy market of country A and country B in Phase I

countries. Once $c+\lambda$ exceeds $\bar{p}_B$, which is the intersection of the demand-curve $D(p_B)$ and the M'_B-curve, the consumption of fossil fuel is too expensive in country B, so that only green energy $\bar{g}_{BB}$ is consumed in country B for all $t \geq T_B$.

For all $t \geq T_B$, the consumer energy price in country A exceeds the one in country B if green energy is not traded. However, the price difference increases in time and may render energy exports profitable for the green energy firm of country B for some $t \in [T_B, T)$. The countries' energy prices satisfy[12]

$$p_A = U'(b^d_A + g^d_A) = M'_A(g^s_{AA}) = c + \lambda, \tag{21}$$

$$p_B = U'(g^d_B) = M'_B(g^s_{BB} + g^s_{BA}), \tag{22}$$

$$p_A \begin{Bmatrix} \leq \\ = \end{Bmatrix} \frac{M'_B(g^s_{BB} + g^s_{BA}) + Q'(g^s_{BA})}{\alpha}, \text{ if } \begin{Bmatrix} g^s_{BA} = 0 \\ g^s_{BA} > 0 \end{Bmatrix}. \tag{23}$$

(21) and (22) govern the allocation of energy consumption and production in country A and B, respectively. They require the energy price in country $i = A, B$ to match the marginal willingness-to-pay for energy $U'(b^d_i + g^d_i)$ of country i's consumer and to match the marginal green energy costs $M'_i(g^s_i)$ in country i.[13] According to (23) energy exports are profitable if country A's energy price p_A covers both country B's marginal green energy costs

[12] (21)-(23) follow from (17) and (18).

[13] In country A the marginal costs of fossil fuel $c+\lambda$ are also equal to the energy price p_A.

$M_B'(g_{BB}^s + g_{BA}^s)$ and the marginal transport costs $Q'(g_{BA}^s)$ related to the share α that is not lost at the conversion. If the inequality sign in (23) holds, transport costs and conversion losses are too high, green energy is not traded and the economy is in Phase II. If the equality sign in (23) holds, the green energy quantity g_{BA}^s is exported from country B to A and Phase III takes place. The point in time at which green energy exports become profitable is denoted by t_a.

The equilibrium of Phase III is illustrated in Fig. 3. In the left panel of Fig. 3 the

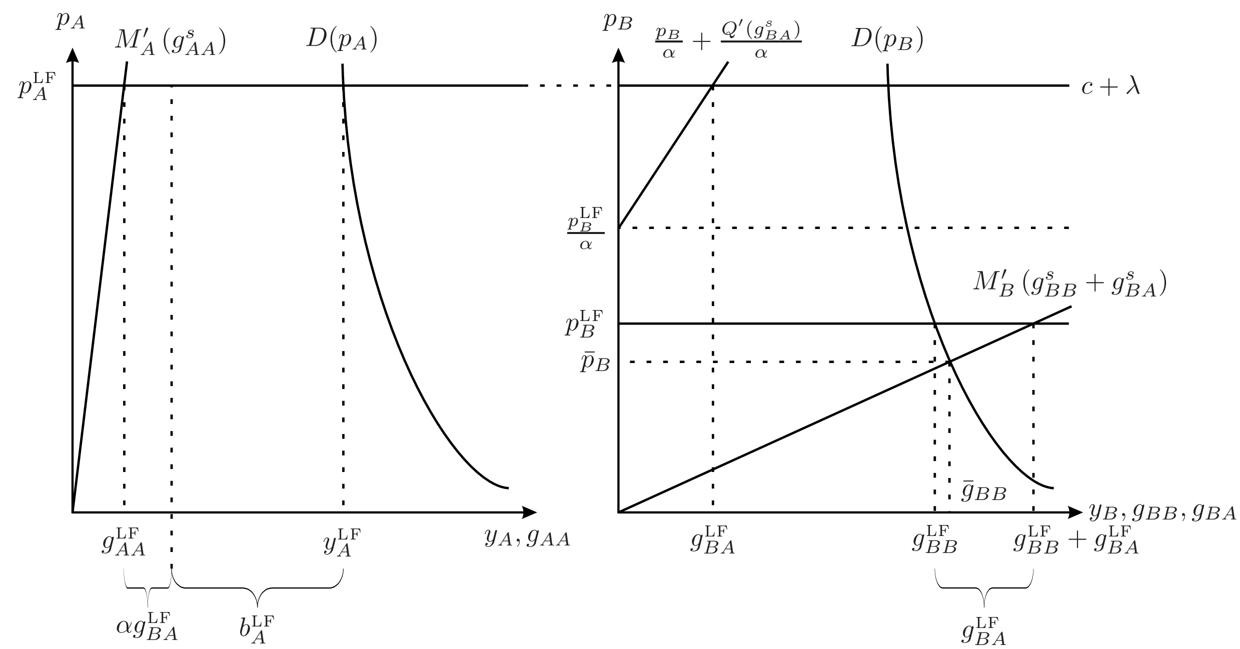


Figure 3: Energy market of country A and country B in Phase III

intersection of the $(c+\lambda)$-line with country A's energy demand curve $D(p_A)$ determines energy consumption y_A^{LF} in A, while the intersection of the $(c+\lambda)$-line with the marginal green energy cost curve M_A' gives domestic green energy production g_{AA}^{LF}. In Phase II, which is not presented in Fig. 3, the difference of total energy consumption and green energy supply equals fossil fuel use $b_A^{\text{LF}} = y_A^{\text{LF}} - g_{AA}^{\text{LF}}$. In Phase III, the green energy quantity $\alpha g_{BA}^{\text{LF}}$ is imported from country B which reduces black energy consumption in country A to the amount $b_A^{\text{LF}} = y_A^{\text{LF}} - g_{AA}^{\text{LF}} - \alpha g_{BA}^{\text{LF}}$.

The right panel of Fig. 3 shows demand and supply in the energy market of country B. In Phase II, country B's energy market clears at the price $\bar{p}_B$ and the green energy quantity $\bar{g}_{BB} = D_B(\bar{p}_B)$. In particular, this equilibrium ensues if the energy price in country A falls short of $\frac{\bar{p}_B}{\alpha}$. Once the increasing scarcity rent has led to $p_A > \frac{\bar{p}_B}{\alpha}$, the economy switches from Phase II to Phase III and energy exports become profitable for the firm of country B.

In Phase III, the intersection of the p_B^{LF}-line with country B's energy demand curve $D(p_B)$ determines green energy consumption g_{BB}^{LF} in country B, and the intersection of the p_B^{LF}-line with the marginal green energy costs curve $M_B'(g_{BB}^s + g_{BA}^s)$ determines the green energy production level $g_{BB}^{\text{LF}} + g_{BA}^{\text{LF}}$ in country B. The amount g_{BB}^{LF} of total production g_B^s is used for domestic consumption, whereas the amount g_{BA}^{LF} is exported to country A.

Because the scarcity rent continuously increases in time, fossil fuel is exhausted at the time T_A and the economy leaves Phase III and enters Phase IV. In that phase, both countries produce green energy only and as in Phase III, green energy is exported from country B to country A. We summarize our results in[14]

Proposition 1 *Suppose that fuel is extracted at $t = 0$. Then, the laissez-faire economy is characterized by*

i) In Phase I, both energy sources are used in country A and country B. The energy prices are given by $p_A^{LF}(t) = p_B^{LF}(t) = c + \lambda(t)$. There is no green energy trade. It holds $\dot{p}_i^{LF}, \dot{g}_{AA}^{LF}, \dot{g}_{BB}^{LF} > 0$ and $\dot{b}_A^{LF}, \dot{b}_B^{LF} < 0$.

ii) In Phase II, fossil fuels are used in country A but not in country B. The energy prices are given by $p_A^{LF}(t) = c + \lambda(t)$ and $\bar{p}_B$, with $\bar{p}_B < p_A^{LF}(t) < \frac{\bar{p}_B}{\alpha}$. There is no green energy trade. Green energy consumption of country B is given by $\bar{g}_{BB}$. It holds $\dot{p}_A^{LF}, \dot{g}_{AA}^{LF} > 0$ and $\dot{b}_A^{LF} < 0$.

iii) In Phase III, fossil fuels are used in country A but not in country B. The energy prices are given by $p_A^{LF}(t) = c + \lambda(t)$ and $p_B^{LF}(t) = \alpha p_A^{LF}(t) - Q'(g_{BA}^{LF}(t))$. Green energy is exported in the amount of $g_{BA}^{LF}(t) > 0$. It holds $\dot{p}_A^{LF}, \dot{p}_B^{LF}, \dot{g}_{AA}^{LF}, \dot{g}_{BA}^{LF} > 0$ and $\dot{b}_A^{LF}, \dot{g}_{BB}^{LF} < 0$.

iv) In Phase IV, the fossil fuel stock is exhausted, only green energy is used in both country A and B, and the energy prices and $g_{BA}^{LF}(t) > 0$ are constant.

The straightforward way of improving our understanding of the role of green energy trade is to compare the results derived in the laissez-faire economy studied so far with those of a laissez-faire economy with national green energy markets and an international black energy market in the otherwise unchanged laissez-faire economy. This scenario is denoted as *green autarky laissez-faire economy* and ensues for infinitely large conversion losses ($\alpha \to 0$). Under green autarky, the Phase I remains unchanged. For all $t \geq T_B$ country B covers its energy consumption by domestic green energy production $\bar{g}_{BB}$ at price $\bar{p}_B$. As time progresses, the marginal fuel cost $c + \lambda$ in country A increase such that fuel extraction

[14]See Appendix A.2 for the proof. Introducing fixed costs of green energy production would lead to an additional Phase 0 in which only fossil fuel is extracted and consumed. That would make the descriptions bulkier without proving any new insights.

reduces and green energy production enhances in country A. For $t \geq T_A$ country A relies only on green energy production $\bar{g}_{AA}$ at the price $\bar{p}_A$ whereby the price and quantity are implicitly defined by $\bar{p}_A = U'_A(\bar{g}_{AA}) = M'_A(\bar{g}_{AA})$. A comparison of the green trade and autarky laissez-faire economy yields

Proposition 2 *In the laissez-faire economy, for all $t \in [t_a, T_A)$ green energy trade ceteris paribus leads to less fossil fuel extraction and more green energy consumption in country A, a higher energy price in country B and less green energy consumption and production in country B.*

The impact of green trade is somewhat related to the green paradox of climate policies. The green paradox arises when climate policies lead to an increase in emissions in the short run (Jensen et al. 2016). Although green trade is no climate policy, policymakers sell the import of green trade as 'climate-friendly' measure. We make the notions of the green trade paradox more precise by introducing

Definition 1

- *An intratemporal paradox [orthodox] of green trade arises, when green trade increases [reduces] fossil fuel extraction in the same period.*
- *An intertemporal paradox of green trade arises, when green trade increases [reduces] fossil fuel extraction in earlier or later periods.*

According to Proposition 2, green energy trade for all $t \in [t_a, T_A)$ ceteris paribus reduces fossil fuel consumption. Compared to the green autarky laissez-faire economy, in the green trade laissez-faire economy the fuel firm's scarcity rent path adapts to a lower demand. That is, the scarcity rent decreases, which lowers the energy price and, therefore, gives rise to more instantaneous fuel consumption. Thus an *intertemporal paradox of green trade* at earlier periods arises.

3.2 Social optimum

In this subsection, we characterize the Pareto optimal intertemporal allocation. The social planner maximizes the sum of intertemporal welfares

$$\int_0^\infty e^{-\rho t} \sum_{i=A,B} \Big[U\left(b_i^d(t)) + g_i^d(t)\right) + \bar{\ell}_i - s_i C(b_A^s(t) + b_B^s(t)) - M_i(g_i^s(t)) - Q(g_{ij}^s(t)) - H(E(t))\Big] \mathrm{d}t, \tag{24}$$

subject to the equation of motions of the limited fossil fuel stock (4), the carbon emissions stock (5), and the resource constraints (9) and (10). In Appendix A.3 we show that the socially optimal allocation is characterized by

$$\begin{aligned} U'(y_A) &= C'(b^s_A + b^s_B) + \kappa + \theta - \zeta_{b^s} - \zeta_{b^d_A} \\ &= M'_A\left(g^s_A\right) - \zeta_{g^s_{AA}} - \zeta_{g^d_A} = \frac{M'_B\left(g^s_B\right)}{\alpha} + \frac{Q'\left(g^s_{BA}\right)}{\alpha} - \frac{\zeta_{g^s_{BA}}}{\alpha} - \zeta_{g^d_A}, \end{aligned} \tag{25}$$

$$\begin{aligned} U'(y_B) &= C'(b^s_A + b^s_B) + \kappa + \theta - \zeta_{b^s} - \zeta_{b^d_B} \\ &= M'_B\left(g^s_B\right) - \zeta_{g^s_{BB}} - \zeta_{g^d_B} = \frac{M'_A\left(g^s_A\right)}{\alpha} + \frac{Q'\left(g^s_{AB}\right)}{\alpha} - \frac{\zeta_{g^s_{AB}}}{\alpha} - \zeta_{g^d_B}, \end{aligned} \tag{26}$$

$$\kappa = \kappa_0 e^{\rho t}, \tag{27}$$

where κ is the scarcity rent of the fossil fuel stock S, and $\theta = \frac{2H'(E)}{\rho+\gamma} = \frac{2h}{\rho+\gamma}$ are the social costs of carbon. The multipliers of the non-negativity constraints $b^s_A, b^s_B, b^d_A, b^d_B, g^d_A, g^d_B, g^s_{AA}, g^s_{AB}, g^s_{BB}, g^s_{BA} \geq 0$ are denoted by $\zeta_{b^s_A} = \zeta_{b^s_B} = \zeta_{b^s}$, $\zeta_{b^d_A}$, $\zeta_{b^d_B}$, $\zeta_{g^d_A}$, $\zeta_{g^d_B}$, $\zeta_{g^s_{AA}}$, $\zeta_{g^s_{AB}}$, $\zeta_{g^s_{BB}}$, and $\zeta_{g^s_{BA}}$. According to the Hotelling-rule (27), the shadow-price of fossil fuels increases with the time preference rate, so that transversality condition (28) implies the complete exhaustion of the fuel stock.

$$\lim_{t\to\infty} e^{-\rho t}\kappa(t)\left[S(t) - S^{opt}(t)\right] \geq 0. \tag{28}$$

Equations (25) and (26) are the allocation rules for efficient energy consumption and production in country A and B, respectively. The following interpretations refer to an interior solution. The equations require that the marginal benefit of energy consumption $U'(y_i)$ in country i equals the marginal social costs of energy production in country i. If fossil energy is consumed in country i, these marginal social costs consist of the sum of the marginal extraction costs C', the scarcity rent κ and the social costs of carbon $\theta = \frac{2h}{\rho+\gamma}$. If green energy is consumed in country i, the marginal costs are the marginal production costs M'_i in case of domestic green energy generation, whereas the marginal costs are composed of the marginal production costs $\frac{M'_j}{\alpha}$ and the marginal transport costs $\frac{Q'}{\alpha}$ in case of energy imports from country $j \neq i$.

In Appendix A.3 we show that the dynamics of fossil fuel extraction and the pattern of green energy flows in the social optimum are analogous to the laissez-faire economy of the previous subsection. The results are summarized in

Proposition 3 *Suppose that fuel is extracted at $t = 0$. Then the social optimum is characterized by*

(i) In Phase I fossil fuel is used in both countries. There are no green energy imports or exports. It holds $\dot{g}^E_{AA}, \dot{g}^E_{BB} > 0$ and $\dot{b}^E_A, \dot{b}^E_B < 0$.

(ii) In Phase II fossil fuel is used in country A but not in country B. There are no green energy imports or exports. It holds $\dot{g}^E_{AA} > 0$ and $\dot{b}^E_A < 0$.

(iii) In Phase III fossil fuel is used in country A but not in country B. Green energy is exported from country B to A. It holds $\dot{g}^E_{AA}, \dot{g}^E_{BA} > 0$ and $\dot{b}^E_A, \dot{g}^E_{BB} < 0$.

(iv) In Phase IV fossil fuel is exhausted. Green energy is exported from country B to A. All quantities are constant.

Comparing the laissez-faire economy with the social optimum reveals that the laissez-faire economy is inefficient due to a climate externality. One way to internalize the externality is to levy a fuel tax at rate

$$\tau(t) = \frac{2H'(E(t))}{\rho + \gamma} = \frac{2h}{\rho + \gamma} \tag{29}$$

in both countries.

4 Unilateral fuel tax: General results

4.1 Non-strategic fuel tax

Next, we analyze the dynamics of the economy when country A uses a climate policy while country B refrains from setting any climate policy. The climate policy at hand is a fuel tax at rate $\tau_A(t)$ levied on country A's consumption of black energy. In this subsection, country A behaves non-strategically and takes the market prices as given.

Country A's unilaterally optimal tax rate follows from maximizing the welfare[15] $\int_0^\infty e^{-\rho t}$ $\{U_A(b^d_A(t) + g^d_A(t)) - p_A(t)(b^d_A(t) + g^d_A(t)) + \bar{\ell}_A + s_A\Pi_F(t) + \Pi_A(t) - H(E(t)) + \tau_A(t)b^d_A(t)\}\mathrm{d}t$ with respect to $b^d_A(t)$ subject to the equation of motions of the carbon emissions stock (5). When doing so, country A neglects its influence on the prices $p^b(t), p_B(t)$, $p^g_A(t)$, $p^g_B(t)$ and the scarcity rent λ. The unilaterally optimal tax rate is characterized by[16]

$$\tau^{\mathrm{U}}_A(t) = \frac{H'(E(t))}{\rho + \gamma} = \frac{h}{\rho + \gamma}. \tag{31}$$

Country A's optimally unilateral fuel tax internalizes the climate externality inflicted on its own residents but leaves uninternalized the externality inflicted on country B's residents. Due to the linearity of the climate damage, the tax rate (31) is constant over time.

[15]The tax changes the budget constraint of the representative consumer of country A into $p_A(t)\left[b^d_A(t) + g^d_A(t)\right] = \omega_A + \Phi(t)$, where $\Phi(t)$ is the lump-sum transfer of tax revenues and

$$p_A(t) = \min\left\{p^b(t) + \tau_A(t), p^g_A(t), \frac{p^g_B(t)}{\alpha}\right\} \tag{30}$$

is country A's energy consumer price. The first-order conditions of the fossil fuel firm and the green energy firms are as in the laissez-faire economy.

[16]See Appendix A.4. We use the superscript U to indicate the unilaterally optimal solution.

Depending on the tax rate τ_A^{U} and the characteristics of the demand and cost functions, the timing is described by one of the 18 cases listed in Lemma **??** of the Online Appendix. Lemma **??** shows that both green energy trade and fuel extraction can end and start again later on. It is beyond the scope of the paper to discuss all possible timings in detail. Therefore, we restrict our discussion to the timings that are the most relevant ones as revealed by our numerical simulations in section 5. These are the timings $0 < T_A < t_c < T_B$ and $0 < t_c < t_d < T_B < T_A$, respectively, where t_c is the point in time country B stops to export its complete green energy production to country A and t_d is the point in time green energy trade ends. At the timing $0 < T_A < t_c < T_B$, which is presented in subsection 4.1.1, green energy trade is prevailing in all phases, whereas at the timing $0 < t_c < t_d < T_B < T_A$, which is presented in subsection 4.1.2, green energy trade stops before the end of fossil fuel use.

4.1.1 Timing $0 < T_A < t_c < T_B$

At early points of time, i.e. for $t \in [0, t_c)$, all green energy produced in country B is exported to country A. For $t \in [0, T_A)$ the energy market equilibrium is illustrated in Fig. 4. Because the price difference $\alpha p_A(t) - p_B(t)$ is large, it is profitable for the green

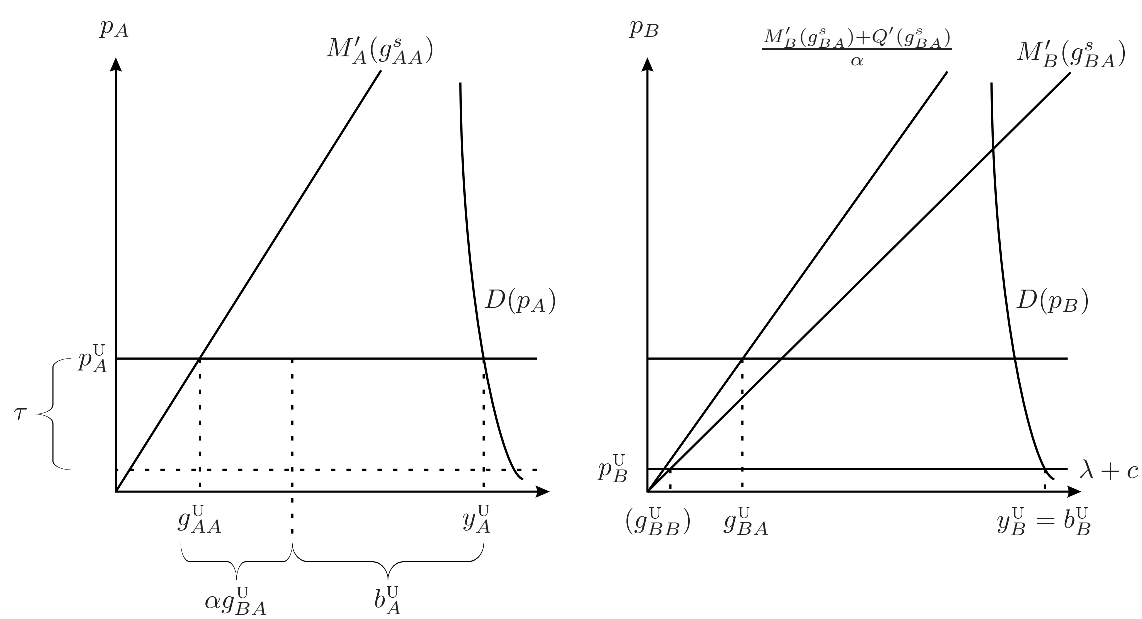


Figure 4: Energy market of country A and country B for $t \in [0, T_A)$

energy producers of country B to export their complete green energy production to country A. Country B covers its own energy consumption y_B^{U} solely by fossil fuel use b_B^{U}. Green energy production in country B is determined by $M_B'(g_{BA}^{\text{U}}(t)) + Q'(g_{BA}^{\text{U}}(t)) = \alpha p_A^{\text{U}}(t)$. In view of the left panel of Fig. 4, country A's energy consumption composes of domestic green energy production g^{U}_{AA}, green energy imports αg^{U}_{BA} and fossil fuel use b^{U}_{A}. Country A's clean energy production is determined by $M'_A(g^{\text{U}}_{AA}(t)) = p^{\text{U}}_A(t)$. Finally, fossil fuel use in country A and B is characterized by $p^{\text{U}}_A(t) = c + \lambda(t) + \tau$ and $p^{\text{U}}_B(t) = c + \lambda(t)$, respectively.

The evolution of green energy production in country A, green energy imports and fossil fuel use follows from differentiating $g^{\text{U}}_{AA}(t)$, $g^{\text{U}}_{BA}(t)$, $b^{\text{U}}_B(t)$ and $b^{\text{U}}_B(t)$ with respect to time, which yields

$$\dot{g}^{\text{U}}_{AA} = \frac{1}{m_A}\dot{\lambda} > 0, \quad \dot{g}^{\text{U}}_{BA} = \frac{\alpha}{m_B + q}\dot{\lambda} > 0, \tag{32}$$

$$\dot{b}^{\text{U}}_A = \left[D' - \frac{1}{m_A} - \frac{\alpha^2}{m_B + q}\right]\dot{\lambda} < 0, \quad \dot{b}^{\text{U}}_B = D'\dot{\lambda} < 0. \tag{33}$$

As time passes, the scarcity rent increases and boosts both clean energy production in country A, the exports to country A and reduces fossil fuel use in both countries.

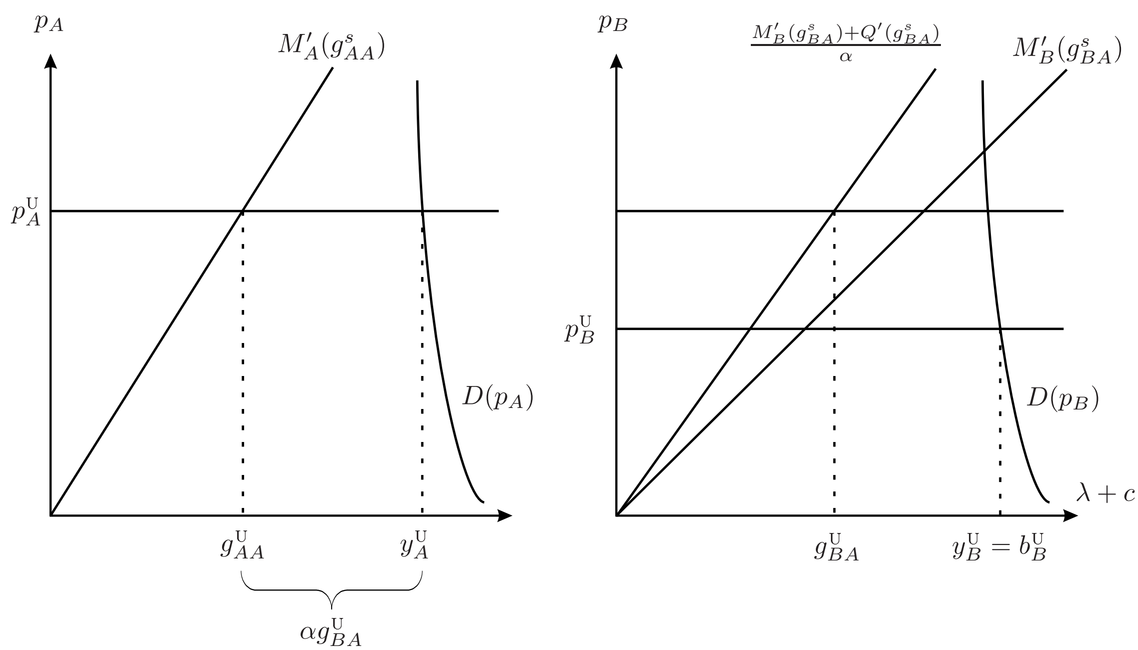


Figure 5: Energy market of country A and country B for $t \in [T_A, t_c)$

At time T_A, the increase of both green energy production in country A and imports from country B together with the increasing price $p^{\text{U}}_A(t)$ drive fossil fuel out of country A's energy market. The associated equilibrium is illustrated in Fig. 5. In country A domestic green energy production g^{U}_{AA} is still determined by $M'_A(g^{\text{U}}_{AA}) = p^{\text{U}}_A$ and imports g^{U}_{BA} are still determined by $M'_B(g^{\text{U}}_{BA}) + Q'(g^{\text{U}}_{BA}) = \alpha p^{\text{U}}_A$, but now the energy price $p^{\text{U}}_A < c + \lambda(t) + \tau$ does not depend on time such that g^{U}_{AA} and g^{U}_{BA} are also time invariant for $t \in [T_A, t_c[$. In country B only fossil fuels are consumed whose consumption reduces over time due to

$p_B^{\text{U}}(t) = c + \lambda(t)$ and an increasing scarcity rent. The price difference $\alpha p_A - p_B(t)$ declines over time.

At time t_c, the price difference $\alpha p_A - p_B(t)$ is so small that it is no longer profitable to export all green energy produced in country B to country A. The market equilibrium is illustrated in Fig. 6. Total green energy production in country B is given by $g_{BA}^{\text{U}}(t) + g_{BB}^{\text{U}}(t) = M_B^{'-1}\left(p_B^{\text{U}}(t)\right)$. The amount $g_{BA}^{\text{U}}(t) = Q^{'-1}\left(\alpha p_A^{\text{U}}(t) - p_B^{\text{U}}(t)\right)$ is exported to country A whereas the amount $g_{BB}^{\text{U}}(t)$ is consumed domestically. Since country B's energy demand is greater than its green energy production for internal consumption, the consumption gap is closed by fossil fuel consumption $b_B^{\text{U}}(t)$. In view of the left panel of Fig. 6, energy consumption of country A is given by $y_A^{\text{U}}(t) = D(p_A^{\text{U}}(t))$ and covered by the amount $g_{AA}^{\text{U}}(t) = M_A^{'-1}\left(p_A^{\text{U}}(t)\right)$ of domestic green energy production and the amount $\alpha g_{BA}^{\text{U}}(t)$ of green energy imports. Differentiating with respect to time yields

$$\dot{g}_{AA}^{\text{U}} = \frac{\alpha}{\phi}\dot{\lambda} > 0, \quad \dot{g}_{BB}^{\text{U}} = \left[\frac{1}{m_B} + \frac{1 - m_A D'}{\phi}\right]\dot{\lambda} > 0, \quad \dot{g}_{BA}^{\text{U}} = -\frac{1 - m_A D'}{\phi}\dot{\lambda} < 0, \tag{34}$$

$$\dot{b}_B^{\text{U}} = \left[D' - \frac{1}{m_B} - \frac{1 - m_A D'}{\phi}\right]\dot{\lambda} < 0, \quad \dot{p}_A^{\text{U}} = \frac{\alpha m_A}{\phi}\dot{\lambda} > 0, \tag{35}$$

with $\phi = \alpha^2 m_A + q - q m_A D'$. The increasing scarcity rent raises the price in country B. Consequently, more green energy and less fossil fuel is consumed in country B. The increasing energy price in country B renders green energy exports to country A less profitable, so that the trade volume decreases, the energy price in country A increases and country A's green energy production is boosted.

Finally, we compare the unilaterally regulated economy analyzed so far with a unilaterally regulated economy under green autarky. In Appendix A.4.2, we prove

Proposition 4 *Suppose that fuel is consumed in both countries at $t = 0$. Then in a unilaterally regulated economy with the timing $0 < T_A < t_c < T_B$, ceteris paribus, the relation of the energy consumption levels with green energy trade and under green autarky is as follows.*

(i) For $t \in [0, T_A)$, green energy consumption is higher and fossil fuel consumption is lower with green energy trade, if $\alpha > 0$ and τ^U is high.

(ii) Consider $t \in [T_A, t_c)$ and fossil fuel use in A without green energy trade. Green energy consumption is higher and fossil fuel consumption is lower with green trade, if α is close to unity, τ^U is small and a is large.

If no fossil fuel is used in A without green energy trade, green energy consumption is lower with green trade if α is close to nil. Fossil fuel use is higher with green energy trade.

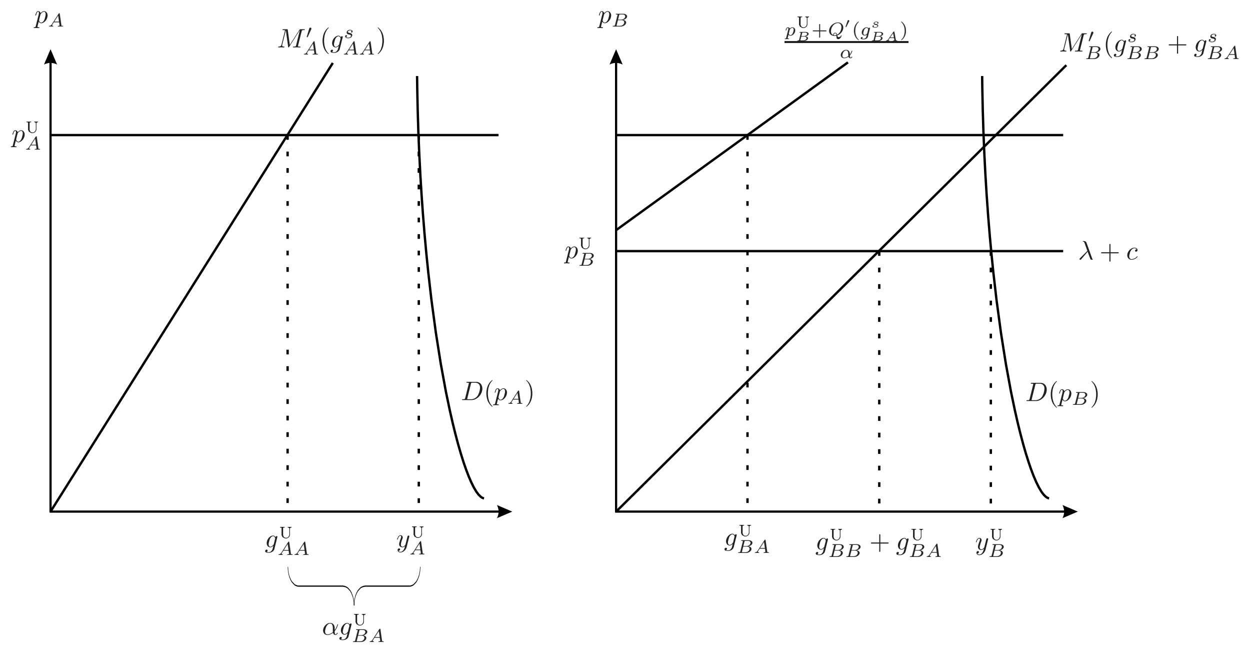


Figure 6: Energy market of country A and country B for $t \in [t_c, T_B)$

(iii) *Consider $t \in [t_c, T_B)$ and fossil fuel use in A without green energy trade. Green energy consumption is lower with green trade. Fossil fuel use is lower with green trade if τ^U is small, α is close to unity and a is large.*

If no fossil fuel is used in A without green energy trade, green energy consumption is lower and fossil fuel consumption is higher with green trade.

To understand Proposition 4 (i), consider Fig. 4. If $\alpha > 0$ and τ^{U} is sufficiently high, the price gap $\alpha p_A(t) - p_B(t)$ is large and induces the green firms of country B to export their complete production to country A with the consequence that in country B only fossil fuel is used. Ceteris paribus, the energy price in country B is the same with and without green energy trade, but the green energy production with green energy trade is larger, because it is completely exported and a fraction of these exports is lost due to conversion. With energy trade the additional production outweighs the conversion losses and green energy production increases compared to the green energy production under green autarky. The imported green energy drives back fossil fuel in the market of country A, while country B replaces the green energy production under autarky by fossil fuels. Since the former effect dominates the latter effect, fuel consumption is lower with green energy trade.

For Proposition 4 (ii) and (iii), we have to distinguish whether fossil fuel is used or not used in country A under green autarky. At first, consider Proposition 4 (ii) and fossil fuel use in A under green autarky. With green energy trade, at $t = T_A$ the green energy exports drive fossil fuel completely out of country A's market and the energy price in A no longer depends on the tax rate but is determined by energy demand and supply of domestic and imported green energy. If α is close to unity and the energy demand parameter a is high, the conversion losses are small and country A's green energy consumption and imports are high. Under green autarky, the price in A is given by $p_A^{\text{AU}} = c + \lambda + \tau^{\text{U}}$, and green energy production is the lower and fossil fuel consumption the higher, the smaller the price, i.e. the smaller the tax rate τ^{U}. With trade, country B only uses fossil fuels and fuel use is is governed by $p_B^{\text{U}}(t) = c+\lambda(t)$. Under green autarky, the energy price reads[17] $p_B^{\text{AU}}(t) = c+\lambda(t)$ and country B uses both fossil fuels and green energy. That is, country B uses more fossil fuels with green trade than under autarky. However, if τ^{U} is small, a is high and α close to unity, green energy exports drive back enough fossil fuels from country A's energy market that green energy production is higher and fossil fuel consumption is lower with green energy trade than under autarky.

In case that country A does not use fossil fuels under green autarky, both under green energy autarky and with green trade country A abstains from consuming fossil fuel. In country B, green energy exports cause a substitution of green energy by fossil fuels such that fossil fuel use is larger with green trade than under green autarky.

Consider now Proposition 4 (iii) and fossil fuel use in A under green autarky. As illustrated in Fig. 6, the price gap $\alpha p_A - p_B$ only renders the exports of some green energy profitable for the green energy firm in country B, and both green energy and fossil fuels are consumed in country B. Ceteris paribus, the energy price in country B is the same with and without green energy trade and hence green energy production in country B is the same with and without green energy trade. With green energy trade, a part of this production is exported to country A. Due to the conversion losses associated to exports, green energy consumption is lower with green energy trade than without. Fossil fuel use is lower with green trade if the amount of fuel driven out of the market of country A is sufficiently large which happens if the tax rate is small, conversion losses are small and energy demand high.

If no fossil fuel is used in country A under green autarky, the trade induced substitution of green energy by fossil fuels in country B leads to more fossil fuel use with green trade ceteris paribus, and we obtain

Proposition 5 *Suppose that fossil fuel is consumed in both countries at $t = 0$ but no fuel is consumed in country A under green autarky for $t \geq T_A$. Then in a unilaterally regulated economy with the timing $0 < T_A < t_c < T_B$ there is an intratemporal green trade paradox for*

[17] The superscript AU stands for autarky.

$t \geq T_A$.

Because of the limited fossil fuel stock, the increase of fossil fuel use in later periods is associated with a reduction of fossil fuel use in early periods. That is, an intratemporal paradox of green trade in late periods goes along with an *intertemporal orthodox* of green trade at early periods.

4.1.2 Timing $0 < t_c < t_d < T_B < T_A$

At this timing again initially (before $t = t_c$), the complete green energy production of country B is exported and both countries use fossil fuels. The energy market equilibrium which is illustrated in Fig. 4 is as for $0 < T_A$ in the timing of the previous subsection. Green energy production in both countries increase in time, while fossil fuel use declines in both countries. Proposition 4 (i) holds and an intratemporal paradox of green energy trade is possible if both α and τ^{U} are small.

As time progresses, the scarcity rent increases and the price gap $\alpha p_A - p_B$ decreases. At $t = t_c$ the price gap has reduced to such an extent that the green energy firm in country B stops exporting its entire green energy production. Then, both countries consume both energy sources.

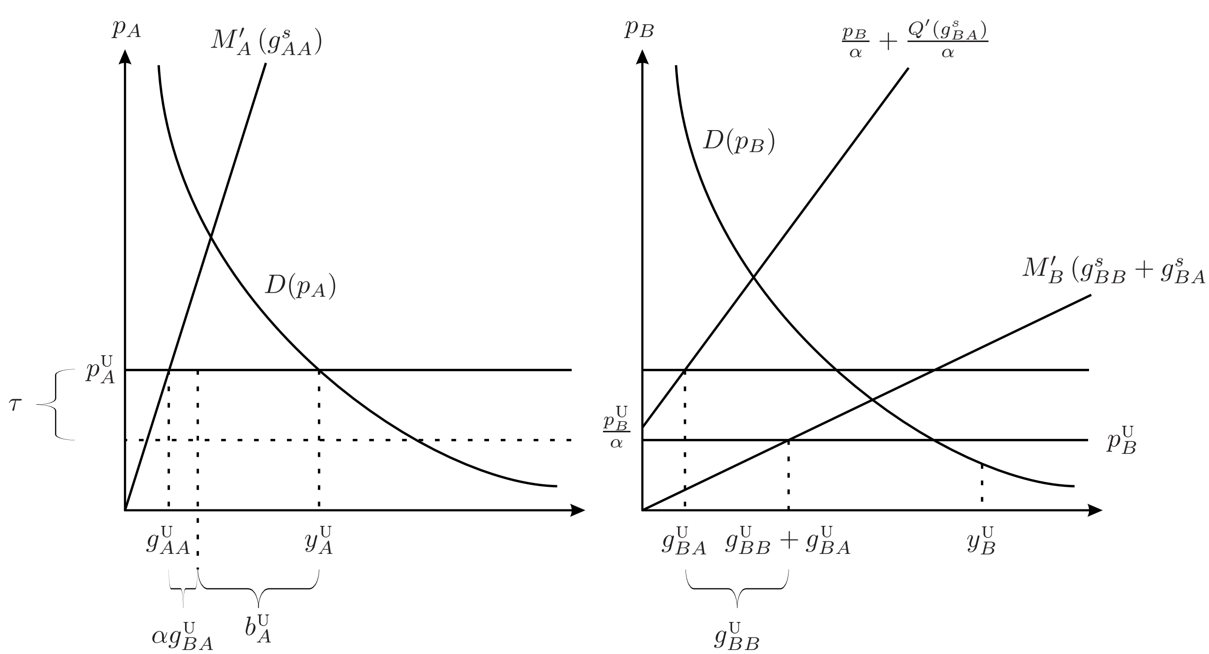


Figure 7: Energy market of country A and country B for $t \in [t_c, t_d)$

For $t \in [t_c, t_d)$, the energy market equilibrium is illustrated in Fig. 7. In country B, energy consumption is y_B^{U} and green energy production is given by $g_{BA}^{\text{U}}+g_{BB}^{\text{U}} = M_B'^{-1}(p_B^{\text{U}})$. The amount $g_{BA}^{\text{U}} = Q'^{-1}(\alpha\tau^{\text{U}} - (1-\alpha)p_B^{\text{U}})$ of country B's green energy production is exported to country A and the amount g_{BB}^{U} of country B's green energy production is consumed domestically. Since country B demands more energy than it produces green energy for internal consumption, the consumption gap is closed by fossil fuel use $b_B^{\text{U}} = y_B^{\text{U}} - g_{BB}^{\text{U}}$. In view of the left panel of Fig. 7, energy consumption of country A is y_A^{U} and covered by domestic green energy $g_{AA}^{\text{U}} = M_A'^{-1}(p_A^{\text{U}})$, green energy imports αg_{BA}^{U} and black energy b_A^{U}.

The evolution of energy production and consumption in country B and energy exports follows from differentiating g_{BA}^{U}, g_{BB}^{U} and b_B^{U} with respect to time, which yields

$$\dot{g}_{BB}^{\text{U}} + \dot{g}_{BA}^{\text{U}} = \frac{\dot{\lambda}}{m_B} > 0, \quad \dot{g}_{BA}^{\text{U}} = -\frac{(1-\alpha)\dot{\lambda}}{q} < 0, \quad \dot{g}_{BB}^{\text{U}} = \frac{\dot{\lambda}}{m_B} + \frac{[1-\alpha]\dot{\lambda}}{q} > 0, \tag{36}$$

$$\dot{b}_B^{\text{U}} = D'\dot{\lambda} - \dot{g}_{BB}^{\text{U}} < 0. \tag{37}$$

The increasing scarcity rent reduces green energy trade and increases green energy production in country B for domestic consumption which overcompensates the decline in exports. Due to an increasing energy price, the higher domestic green energy consumption also reduces fuel use in country B.

With respect to country A, differentiation leads to

$$\dot{g}_{AA}^{\text{U}} = \frac{\dot{\lambda}}{m_A} > 0, \quad \dot{g}_{AA}^{\text{U}} + \alpha\dot{g}_{BA}^{\text{U}} = \left[\frac{1}{m_A} - \alpha\frac{(1-\alpha)}{q}\right]\dot{\lambda}, \tag{38}$$

$$\dot{b}_A^{\text{U}} = \left[D' - \frac{1}{m_A} + \frac{\alpha[1-\alpha]}{q}\right]\dot{\lambda}. \tag{39}$$

As in case of country B, the increasing scarcity rent also boosts green energy production in country A. The evolution of both green energy consumption and fossil fuel consumption in country A depends on the conversion loss parameter α, the production cost parameter m_A and the transport cost parameter q.

If there are almost no conversion losses ($\alpha \to 1$) or if green energy production costs in country A are very low ($m_A \to 0$), the increasing domestic green energy production overcompensates lower imports and pushes fossil fuels out of the market. In contrast, with high transport costs ($q \to \infty$) or high conversion losses ($\alpha \to 0$), energy imports are almost nil and their evolution has, therefore, only a marginal impact. However, if conversion losses are mediocre ($\alpha \approx 0.5$) and transport costs are small ($q \to 0$), a reduction of green energy imports causes an increase of fuel consumption in country A.

At time t_d, the price difference is so small that country B stops exporting green energy to country A. For $t \in [t_d, T_B)$ the energy market equilibrium is illustrated in Fig. 8. Energy

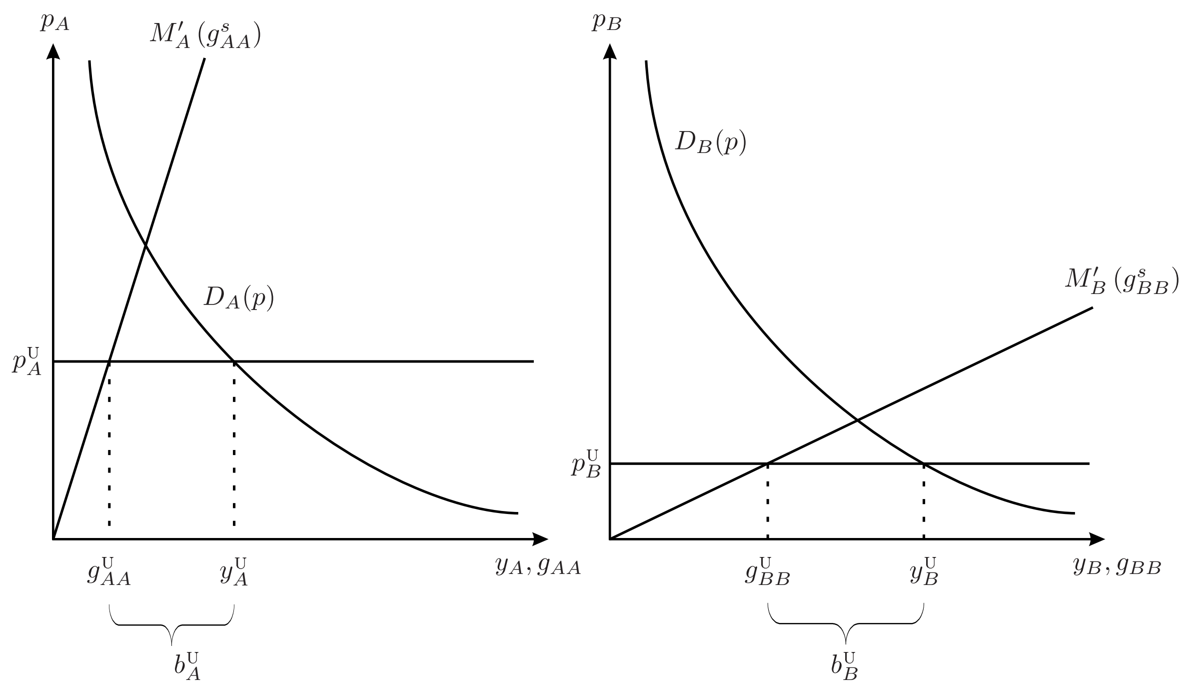


Figure 8: Energy market of country A and country B for $t \in [t_d, T_A)$

consumption of country $i = A, B$ is y^{U}_i and consists of domestic green energy production $g^{\mathrm{U}}_{ii} = M_i'^{-1}(p^{\mathrm{U}}_i)$ and black energy b^{U}_i. From

$$\dot{g}^{\mathrm{U}}_{ii} = \frac{\dot{\lambda}}{m_i} > 0, \quad \dot{b}^{\mathrm{U}}_i = D'\dot{\lambda} - \dot{g}^{\mathrm{U}}_{ii} < 0 \tag{40}$$

we infer that fuel use reduces and green energy enhances over time due to increasing energy prices in both countries driven by an increasing scarcity rent. However, the price difference $\alpha p_A - p_B$ still declines, so that green energy is not traded.

With the considered timing $0 < t_c < t_d < T_B < T_A$, country B ends fossil fuel use before country A. Thus for $t \in [T_B, T_A)$ fuel is only used in country A and the energy market evolves according to

$$\dot{g}^{\mathrm{U}}_{AA} = \frac{\dot{\lambda}}{m_A} > 0, \qquad \dot{b}^{\mathrm{U}}_A = D'\dot{\lambda} - \dot{g}^{\mathrm{U}}_{AA} < 0. \tag{41}$$

Fuel use in country A ends at time T_A.

Finally, we compare the unilaterally regulated economy analyzed so far with a unilaterally regulated economy under green autarky. In Appendix A.4.3, we prove

Proposition 6 *Suppose that fossil fuel is consumed in both countries at $t = 0$. Then in a unilaterally regulated economy with the timing $0 < t_c < t_d < T_B < T_A$, ceteris paribus, the relation of the energy consumption levels with green energy trade and under green autarky is as follows.*

(i) For $t \in [0, t_c)$, green energy consumption is higher and fuel consumption is lower with green energy trade if $\alpha > 0$ and τ^U is large.

(ii) For $t \in [t_c, t_d)$, green energy consumption is lower and fuel consumption is higher with green energy trade.

(iii) For $t \in [t_d, T_B)$, the consumption levels are identical.

Proposition 6 (i) is identical to Proposition 4 (i) and can be explained in the same way. To understand Proposition 6 (ii), consider Fig. 7. Recall that the price gap $\alpha p_A - p_B$ is such that some but not all of country B's produced green energy is exported to country A. Therefore, both green energy and fossil fuels are consumed in country B. Ceteris paribus, country B's energy price and country B's green energy production is the same with and without green energy trade. With green energy trade, a part of this production is exported to country A. Due to the conversion losses associated to exports, green energy consumption is lower with green energy trade than without. To substitute the export-induced loss of green energy consumption, more fossil fuels are consumed with green energy trade and we get

Proposition 7 *Suppose that fossil fuel is consumed in both countries at $t = 0$. Then in a unilaterally regulated economy with the timing $0 < t_c < t_d < T_B < T_A$ there is an intra-temporal green trade paradox for $t \in [t_c, t_d)$.*

Because of the limited fossil fuel stock, the increase of fossil fuel use between t_c and t_d causes an increase of the scarcity rent, which in turn reduces fossil fuel use either in earlier or in later periods. That is, an intratemporal paradox of green trade between t_c and t_d goes along with an *intertemporal orthodox* of green trade either at earlier or at later periods.

4.2 Strategic fuel tax

In this subsection, we relax the assumption that the government of country A is a price-taker. Rather, we assume that it considers its influence on equilibrium quantities and prices and, therefore, acts strategically. Now the government does not only use the fuel tax to internalize the climate damages within its country but also to manipulate the market equilibrium to the benefit of country A. Following Eichner et al. (2023), we consider a form of a Stackelberg game, where country A is the leader and firms and consumers are followers.[18]

[18]Our approach implies that country A is committed to its strategy chosen at $t = 0$. That is, we assume open-loop strategies. This approach is also used by Lewis and Schmalensee (1980), Benchekroun et al. (2009,2010) and Battaglini and Harstad (2016). The time consistency problem of the approach is discussed by Eichner et al. (2023).

In Appendix A.5, we show that the unilateral fuel tax of a strategically acting country A satisfies

$$\tau_A^{\text{ST}}(t) = \frac{H'(E(t))}{\rho+\gamma} + e^{\rho t}\text{SE} = \frac{h}{\rho+\gamma} + e^{\rho t}\text{SE} \tag{42}$$

where

$$\begin{aligned}\text{SE} = \frac{1}{-\int_0^T e^{\rho j}\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj}\Bigg\{\alpha\int_0^T\left[g_{BA}^s(j)\frac{\partial p_A(j)}{\partial\lambda(j)} - g_{AB}^s(j)\frac{\partial p_B(j)}{\partial\lambda(j)}\right]dj \\ + \left[\int_0^T b_A^d(j)dj - s_A S_0\right] + \frac{h}{\rho+\gamma}\int_0^T\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj\Bigg\}.\end{aligned} \tag{43}$$

The strategic effects SE compose of two *terms-of-trade effects* and one *emission effect*.[19]

(1) The *terms-of-trade effect* related to fossil fuels is given by $\frac{\int_0^T b_A^d(j)dj - s_A S_0}{-\int_0^T e^{\rho j}\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj}$. If country A is an importer of fossil fuels, i.e. its fuel consumption exceeds its share of the fuel endowment, the terms-of-trade effect is positive and induces country A to increase its fuel tax, which lowers fuel demand, depresses the energy price and, therefore lowers the country's fuel import costs. If country A exports fossil fuels the preceding effects are reversed.

(2) The *terms-of-trade* effect related to green energy is given by $\frac{\alpha\int_0^T\left[g_{BA}^s(j)\frac{\partial p_A(j)}{\partial\lambda(j)} - g_{AB}^s(j)\frac{\partial p_B(j)}{\partial\lambda(j)}\right]dj}{-\int_0^T e^{\rho j}\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj}$. If country A imports green energy ($g_{BA}^s(j) > 0, g_{AB}^s(j) = 0$), the terms-of-trade effect is positive and induces country A to increase the fuel tax, which lowers the energy price and, therefore, the green energy import costs. If country A is exporter of green energy ($g_{BA}^s(j) = 0, g_{AB}^s(j) > 0$), the preceding incentives are exactly reversed.

(3) The *emission effect* $\frac{h}{\rho+\gamma}\frac{\int_0^T\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj}{-\int_0^T e^{\rho j}\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj}$ is negative and induces country A to reduce its fuel tax, which increases the energy price, lowers fuel consumption in country B, and reduces carbon leakage to country B.

There are many possible timings in an economy with a unilaterally strategic fuel tax which are characterized in Lemma **??** of the Online Appendix. We refrain from analyzing these timings in detail and turn in the next section to a calibration.

5 Unilateral fuel tax: Calibration

To further study the effect of green energy trade on energy production and consumption, we calibrate our model to the world oil market in 2030. Because power-to-liquid technologies

[19] The following interpretations are based on the assumptions $-\frac{\partial b_B^d(j)}{\partial\lambda(j)} > 0$, $\frac{\partial p_A(j)}{\partial\lambda(j)} < 0$ and $\frac{\partial p_B(j)}{\partial\lambda(j)} < 0$.

allow to produce perfect substitutes of oil, we choose e-fuels as green energy. E-fuels do not need to be converted for transportation ($\alpha = 1$).[20] According to EIA (2021), the world oil reserves were 1735 billion barrels in 2019, with an annual production of 30.5 billion barrels or 51586 TWh, respectively. Consequently, the forecasted oil stock in 2030 is $S_0 = 2,438,832.56\,\text{TWh}$. The annual production gives rise to 13,172 million tons of CO_2 implying a factor of 255,340.5963 tons CO_2 per TWh.

To divide the world into two countries, A and B, we proceed in two steps. First, we form nine regions as in Galimova et al. (2023), which are Europe, Eurasia, Northeast Asia (NE Asia), Southeast Asia (SE Asia), Middle East and Northern Africa (MENA), North America, South America, Sub-Sahara Africa (SSA), and the South Asian Association for Regional Cooperation (SAARC).[21] According to the simulations of Galimova et al. (2023), Europe, Eurasia and NE Asia will import green energy in 2050, while SSA, S. America, N. America and MENA will be net-exporter. SAARC and SE Asia will neither import nor export larger quantities of green energy. To get two countries of almost identical size, we let Europe, Eurasia, NE Asia and SE Asia form country A and the remaining regions form country B. Using data from BP (2021) shows that 10.3% of the world oil reserves are located in country A and the remaining 89.7% in country B.

Currently, the countries' demand for liquid fuels is almost completely covered by oil. According to projections of Galimova et al. (2023), e-fuels will have a market share of only 0.01% in 2030. Neglecting this small share, using the initial price elasticity of $\epsilon(0) = 0.5$ from Labandeira et al. (2017, p. 553) and setting the oil price to 80 \$ per barrel we determine the parameter of the demand function and obtain $z = 0.000274\,\frac{\text{TWh}}{\$}$ and $a = 38,689.5\,\text{TWh}$.

Following Kollenbach and Schopf (2022) and Eichner et al. (2023), the time-preference rate is set $\rho = 3\%$ and the decay rate $\gamma = 1.44\%$. The latter ensures that the share of one CO_2 unit remaining in the atmosphere after 50 years is identical with the share calculated by Joos et al. (2013). We use the current EU-ETS price of about 80 \$ per ton of CO_2 as indicator for the social costs of carbon, which yields $h = 453,484.899\ \frac{\$}{\text{TWh}}$.

Power-to-liquid (and also power-to-gas) technologies are still at an early development stage and hardly utilized on a large, industrial scale. Therefore, any calibration based on the current technology level and related production costs underestimates the potential of these technologies. To take account of future technological improvements, we calibrate our model to the projections of Galimova et al. (2023), which estimate the production costs of e-fuels

[20] To determine the effect of trade in the presence of conversion losses, we set $\alpha = 0.8$ in subsection 5.3.

[21] See also Tab. 1 in the Appendix.

at about 105 $\frac{\$}{\text{MWh}}$ in country A and at about 73.5 $\frac{\$}{\text{MWh}}$ in country B in 2050. The associated production levels are $2,677\,\text{TWh}$ in country A and $7,416\,\text{TWh}$ in country B which yield production costs of $281,085,000,000\,\$$ and $545,076,000,000\,\$$, respectively. Making use of these costs in (7) we obtain $m_A = 78,446.0217\,\frac{\$}{\text{TWh}^2}$ and $m_B = 19,822.0065\,\frac{\$}{\text{TWh}^2}$.

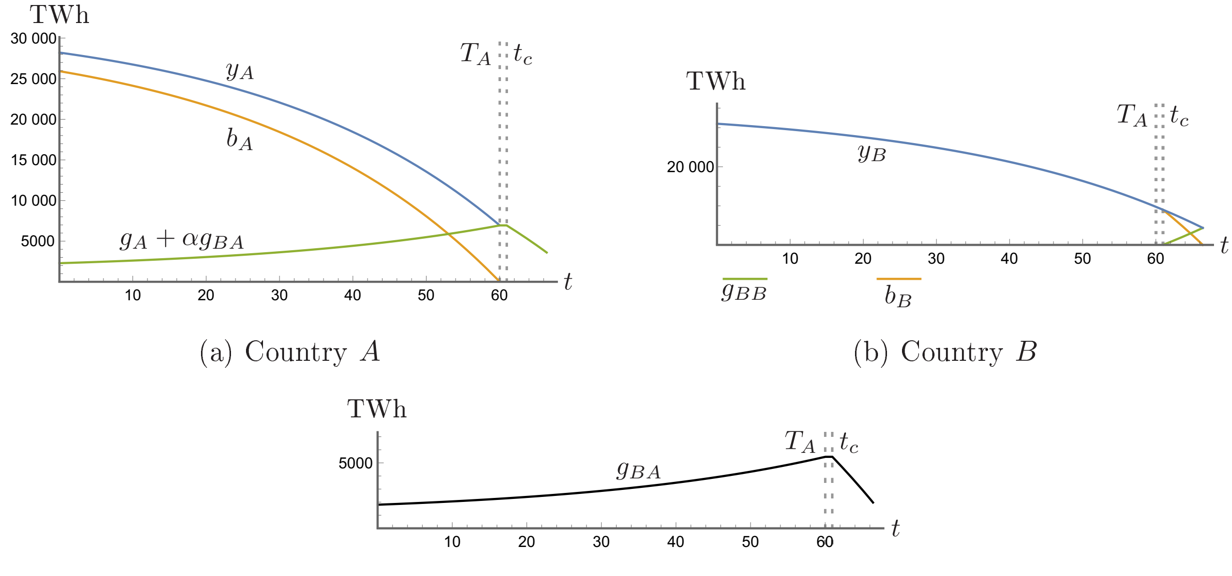
For the transportation costs of e-fuels, Galimova et al. (2023) assume 1.5 $\frac{€}{\text{MWh}}$. For exports in the amount of $2,278\,\text{TWh}$ the transportation costs are about $3,587,850,000\$$. Accounting for the transportation cost in (8) and solving with respect to q we get $q = 1,382.7919\,\frac{\$}{\text{TWh}^2}$.

5.1 Non strategic unilateral fuel tax

When country A does not act strategically, the timing in the calibrated economy is $0 < T_A < t_c < T_B$. For the description of the energy market equilibria we refer to subsection 4.1.1. The energy production and consumption paths of the partly regulated economy with green energy trade and under green autarky are illustrated in Fig. 9 and Fig. 10, respectively. With trade, fossil fuel use in country A will end at $T_A = 60.0205$. During this time, total energy consumption decreases from $y_A(0) = 28,214.9\,\text{TWh}$ to $y_A(T_A) = 6,941.35\,\text{TWh}$. Initially, oil covers 91.88% of energy demand, domestic e-fuel production 1.73% and e-fuel imports 6.39%. For country B, initial energy consumption is $y_B(0) = 31,013.5\,\text{TWh}$, with a fossil fuel share of 100%, because country B exports its complete e-fuel production to country A until time $t_c = 60.9597$. Afterwards, domestic e-fuel consumption increases to reach its maximal and steady-state level of $4,350.88\,\text{TWh}$ at $T_B = 66.415$, the time oil use ends in country B. Green energy trade increases from $g_{BA}(0) = 1,802.81\,\text{TWh}$ to its maximum of $g_{BA}(t) = 5,464.29\,\text{TWh}$, which holds for $t \in [T_A, t_c)$. After country B also starts to consume e-fuels at time t_c, trade decreases to its steady-state level of $g_{BA}(t) = 1,971.56\,\text{TWh}$ for $t \geq T_B$.

Under green autarky, country A consumes $y_A(0) = 28189.3\text{TWh}$ initially, with an oil share of 98.27%. In country B, fossil fuels cover 95.42% of the initial energy consumption of $y_B(0) = 30,987.8\,\text{TWh}$. The oil share decreases in both countries. At $T_A = 66.0312$, oil consumption ends in country A and e-fuels meet the complete demand of $y_A(t) = 1,719.98\,\text{TWh}$. In country B, oil consumption vanishes at $T_B = 64.3647$ and its steady-state (green) energy consumption is $y_B(t) = 6,015.88\ \text{TWh}$.

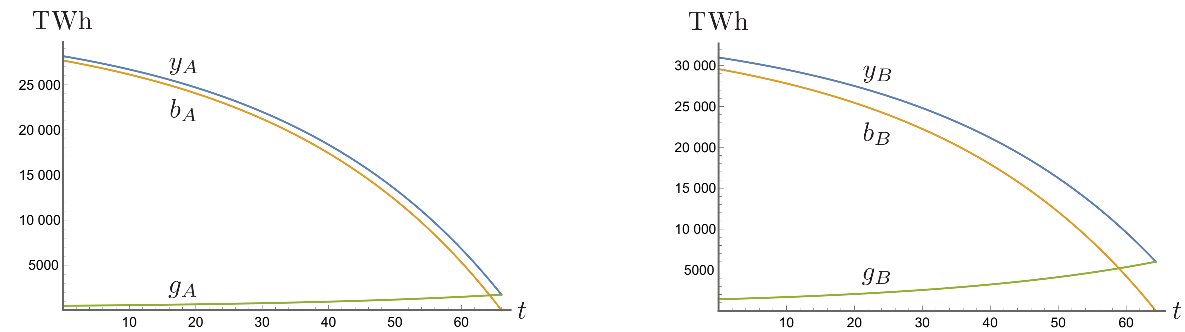
As illustrated in Fig. 11, the fossil fuel extraction path with green energy trade is located below the one for green autarky at early periods. With green energy trade, initial



(a) Country A (b) Country B

(c) Green energy trade

Figure 9: Energy consumption and production with unilateral policy and green trade, without strategic behavior



(a) Country A (b) Country B

Figure 10: Energy consumption and production with unilateral policy under green autarky, without strategic behavior

extraction is 332.3 TWh lower than without green energy trade. The extraction gap decreases until the extraction paths intersect at time $t = 46.9117$. Fig. 11 illustrates the intertemporal orthodox paradox of green trade at early periods and the intratemporal paradox of green trade at late periods.

5.2 Strategic unilateral fuel tax

When the government of country A uses climate policy to manipulate prices and amounts in its favor, the strategic effect is positive, i.e. the terms-of-trade effects dominate the emission effect. As in case of no strategic behavior, the timing is given by $0 < T_A < t_c < T_B$.

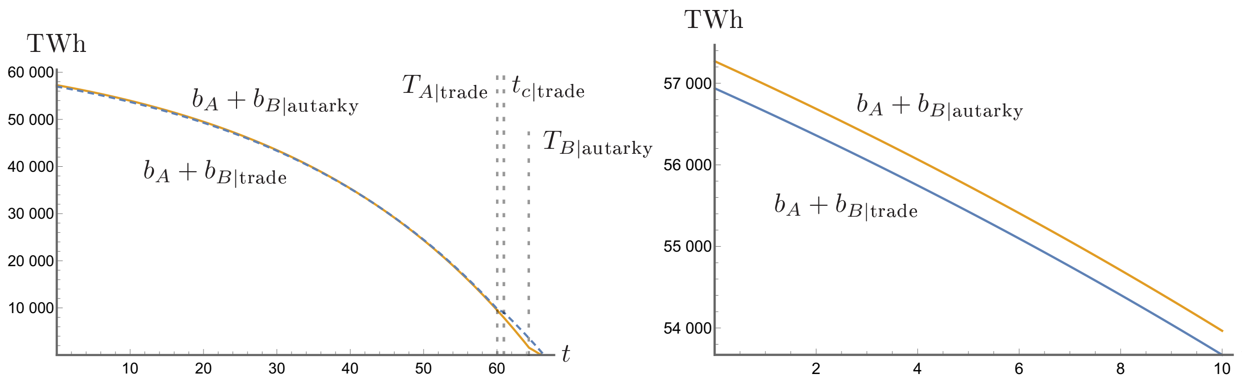
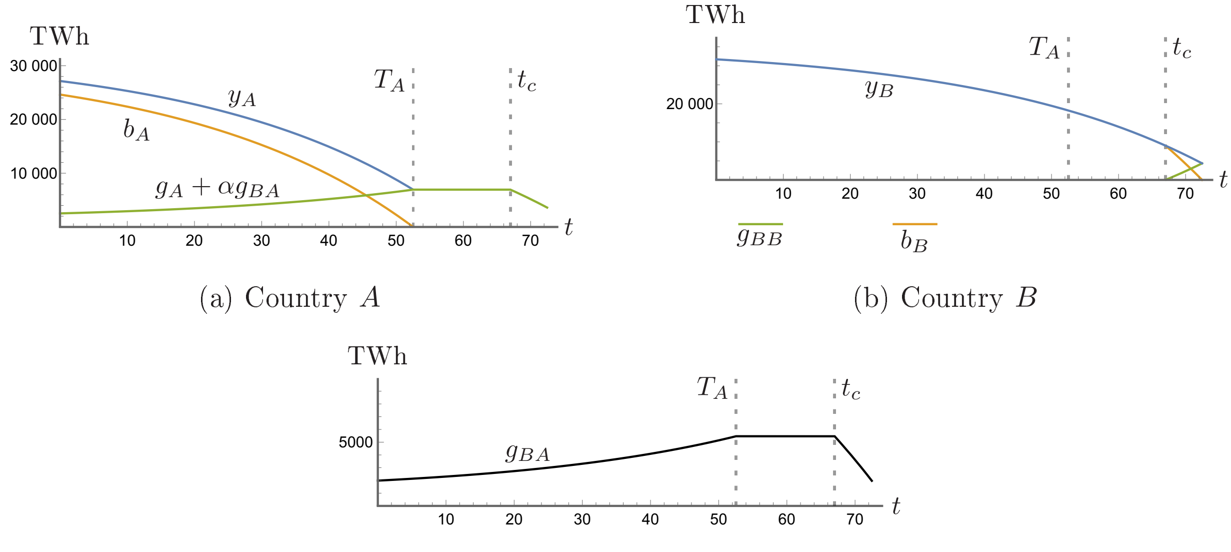


Figure 11: Fossil fuel extraction paths without strategic behavior

However, the time fossil fuel use ends in country A is antedated to $T_A = 52.5296$, while both the period of complete exportation of country B's e-fuel production and the fossil fuel utilization period of country B are extended to $t_c = 67.0012$ and $T_B = 72.4565$, respectively. The energy consumption and production paths with green energy trade are illustrated in Fig. 12. Initially, country A consumes $y_A(0) = 27,154.2$ TWh. The oil share is 90.71%, whereas imported e-fuels account for 7.31% and domestically produced e-fuels for 1.98%. Compared to the case without strategic behavior, country A uses less energy but with higher shares of domestic and imported e-fuels. Oil consumption continuously decreases until it is abandoned at time T_A. Between T_A and t_c, country A's energy consumption is constant and given by $y_A(t) = 6941.35$ TWh. With $g_{BA}(t) = 5464.29$ or a share of 78.72%, imported e-fuels account for the majority of consumption. After country B starts to consume e-fuels, energy consumption in country A declines until it reaches its steady-state level of $y_A(T_B) = 3,603.89$ TWh, with an import share of 54,71%. Because no oil is used after T_A in country A, the strategic effects do not affect the green energy consumption and production for $t \geq T_A$. In case of country B, initial energy consumption reads $y_B(0) = 31,711.3$TWh, which is almost 700 TWh higher than without strategic behavior of country A. As in case of subsection 5.1, energy consumption decreases and country B starts to use e-fuels after time t_c.

Under green autarky, we still find a positive strategic effect, which reduces the fossil fuel utilization period in country A but extend the corresponding period in country B. Fuel use vanishes in country A at $T_A = 58.8342$ and in country B at $T_B = 70.8802$. The corresponding energy consumption and production paths are illustrated in Fig. 13. Initially, country A consumes $y_A(0) = 27,168.8$ TWh, which is about 1000 TWh lower than without strategic action, while energy consumption of country B is increased by 726.3 TWh to $y_B(0) = 31,739.8$ TWh.

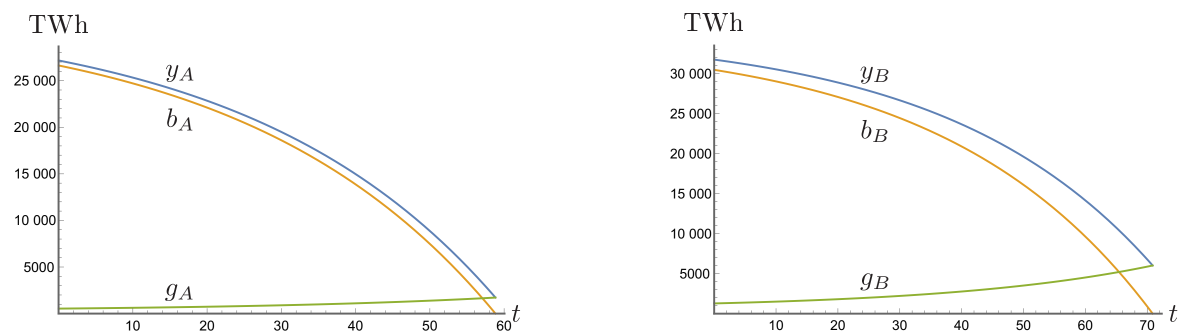


(a) Country A

(b) Country B

(c) Green energy trade

Figure 12: Energy consumption and production with unilateral policy, green trade, and strategic behavior of country A



(a) Country A

(b) Country B

Figure 13: Energy consumption and production with unilateral policy and strategic behavior of country A under green autarky

For the extraction path, we find that green energy trade shifts oil extraction from early periods to late ones. That is, the extraction path is flattened. As illustrated in Fig. 14, initial extraction is reduced by 749.5 TWh to $b_A(0)+b_B(0) = 56,343.4$ TWh by green energy trade. The extraction with trade stays below the one for autarky until $t = 54.6443$. Thus, compared with the case of no strategic behavior, both the reduction of initial extraction and the time of intersection are boosted. However, the qualitative findings with respect to the intertemporal orthodox of early periods and the intratemporal paradox at late periods do not change through strategic behavior.

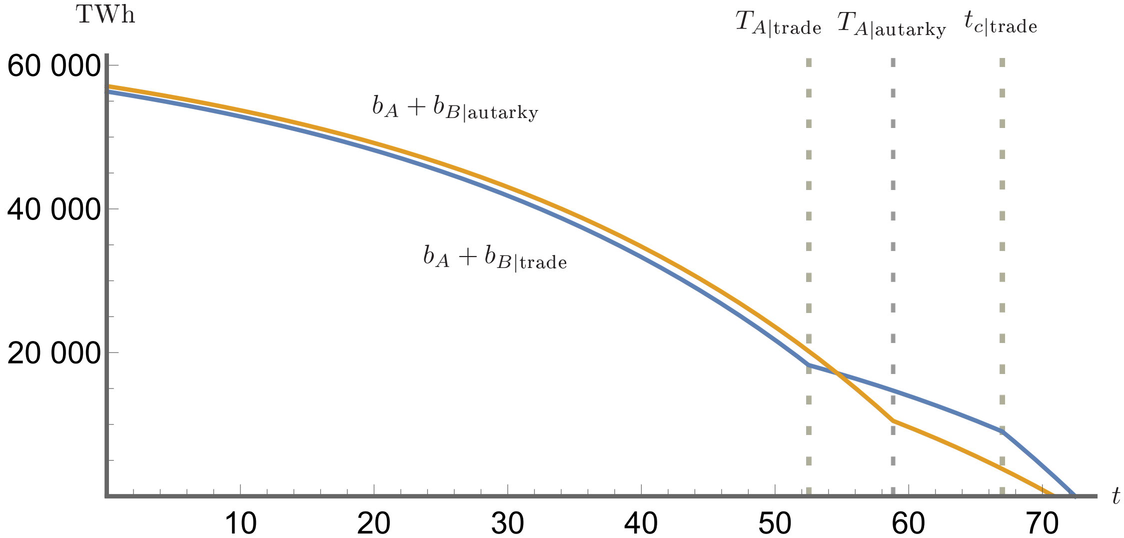


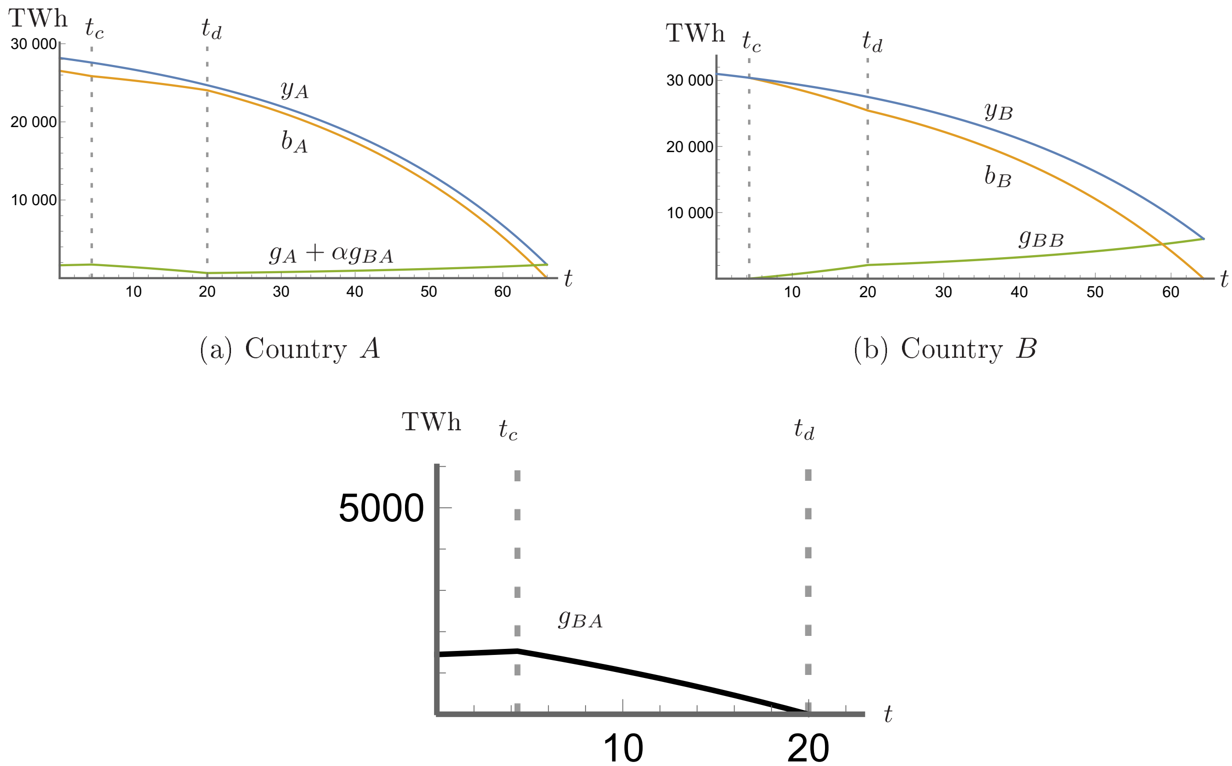
Figure 14: Fossil fuel extraction paths with strategic behavior

5.3 Conversion losses

In this subsection, we investigate the impact of conversion losses. For that purpose we consider the calibrated economy of the previous subsection but now assume that 20% of traded green energy is lost due to conversion, i.e. we replace $\alpha = 1$ by $\alpha = 0.8$.

Without strategic behavior, the timing is given by $0 < t_c < t_d < T_B < T_A$. For the description of the energy market equilibria we refer to subsection 4.1.2. Fig. 15, illustrates energy consumption and production with green energy trade, while Fig. 10 still illustrates energy consumption and production under green autarky. The corresponding extraction paths are depicted in Fig. 16. First, due to conversion losses green energy trade ends at $t_d = 19.9787$ and trade is no longer profitable in the long-run. In early periods, i.e. until $t_c = 4.336$, country B still exports its complete e-fuel production, but domestic e-fuel consumption starts much earlier than with $\alpha = 1$. The fossil fuel consumption period is shortened in country B and ends now at $T_B = 64.2982$. In contrast, in country A the fossil fuel consumption period is extended and ends at $T_A = 65.9647$.

The comparison of the extraction paths shows that at early periods fuel use is higher with green energy trade than without trade. That is, the result from the case for $\alpha = 1$ is reversed and there arises an intratemporal paradox. In particular, initial fuel consumption with green trade is with the quantity $b_A(0)+b_B(0) = 57513.8$ TWh higher than the initial fuel consumption under autarky with the quantity $b_A(0)+b_B(0) = 57270.6$ TWh. Extraction with green energy trade remains higher than under green autarky until $t = 18.6048$. Interestingly, there emerges an intratemporal paradox not only at early periods but also at mean periods.



(a) Country A

(b) Country B

(c) Green energy trade

Figure 15: Energy consumption and production with unilateral policy, green trade and $\alpha = 0.8$, without strategic behavior

If the government of country A acts strategically, trade ends before oil is exhausted and the timing is $0 < T_A < t_c < t_d < T_B$. Until $t_c = 59.9096$, country B exports its complete e-fuel production to country A, which is about 7 periods earlier than in case of $\alpha = 1$. After t_c, country B starts to use e-fuels and ends its exports completely at $t_d = 66.3102$. In contrast, trade does not end as with $\alpha = 1$. Fuel use ends in country A at $T_A = 54.2802$ and in country B at $T_B = 70.0478$. The energy production and consumption paths are illustrated in Fig. 17.

Finally, Fig. 18 compares the extraction path with green energy trade with the path for green autarky. As in case of $\alpha = 1$, the path with trade is located below the autarky-path at early periods and the paths intersect approximately at time $t = 54$. However, the gap is smaller. Initially, fuel use with green trade reads $b_A(0)+b_B(0) = 56,924.5$ TWh, while under autarky we find $b_A(0) + b_B(0) = 57,092.9$ TWh. The gap is reduced by around 580 TWh. Compared to $\alpha = 0.8$ and non-strategic behavior, the results of strategic action are remarkable. Strategic behavior flattens the extraction path and reverses the intratemporal paradox at early periods into an orthodox.

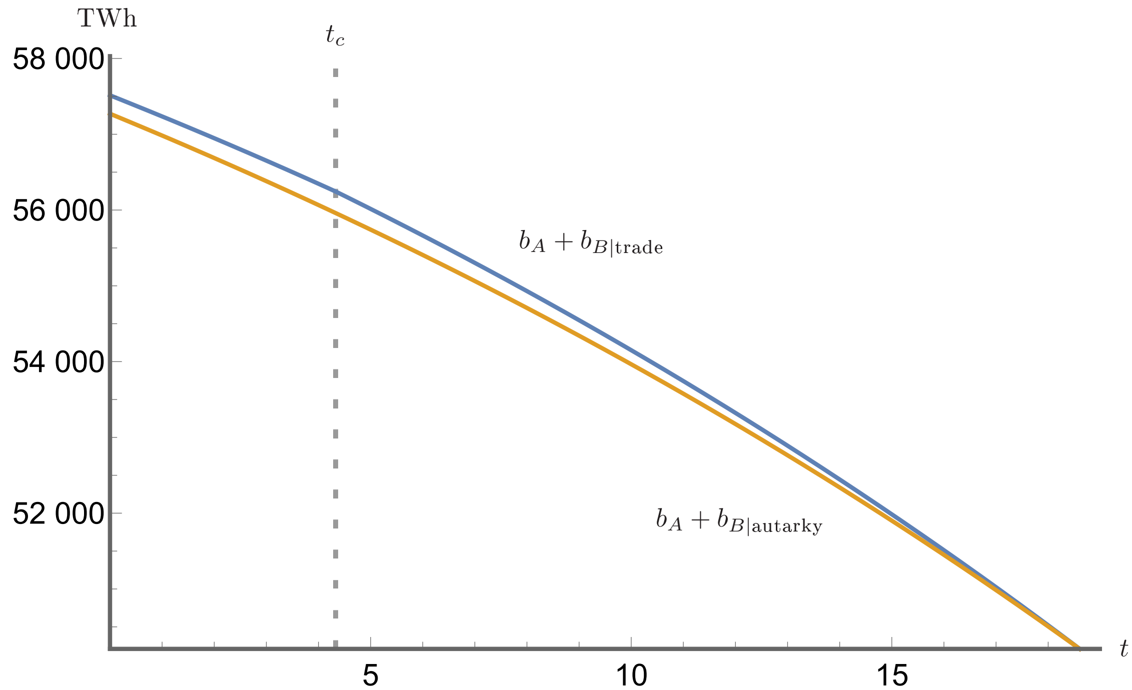


Figure 16: Fossil fuel extraction paths without strategic behavior for $\alpha = 0.8$ and $t \in [0, 19)$

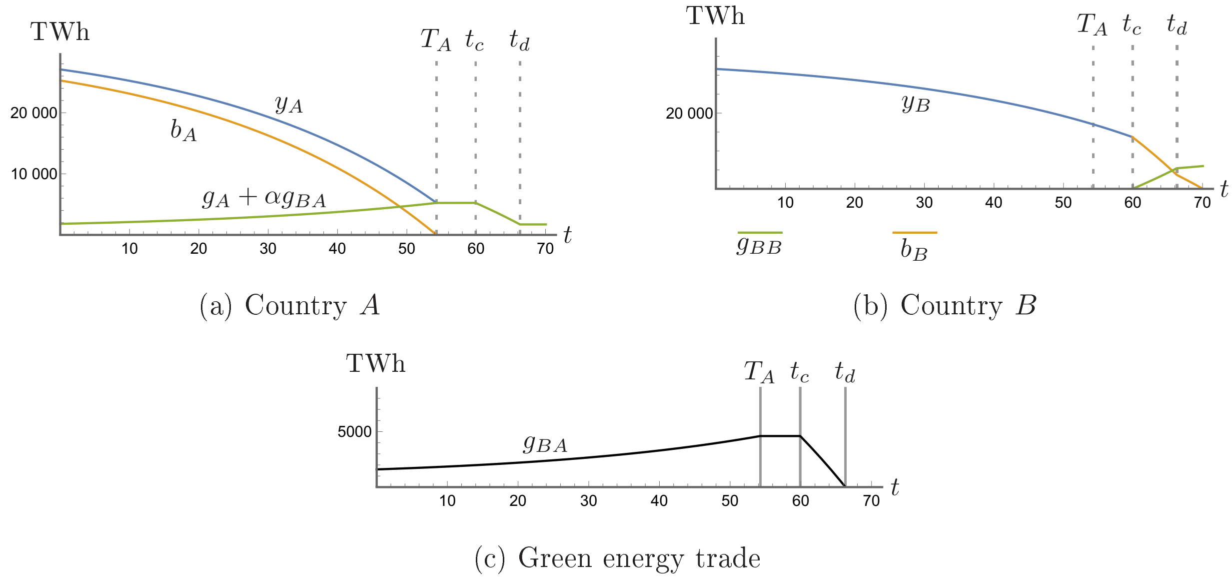


Figure 17: Energy consumption and production with unilateral policy, green trade, $\alpha = 0.8$, and strategic behavior of country A

6 Concluding remarks

In the present paper, we have analyzed the effects of green energy trade on fossil fuel extraction in a dynamic two-country Hotelling model. Opening borders for green energy trade may flatten or steepen the extraction path and may lead to inter- and intratemporal para-

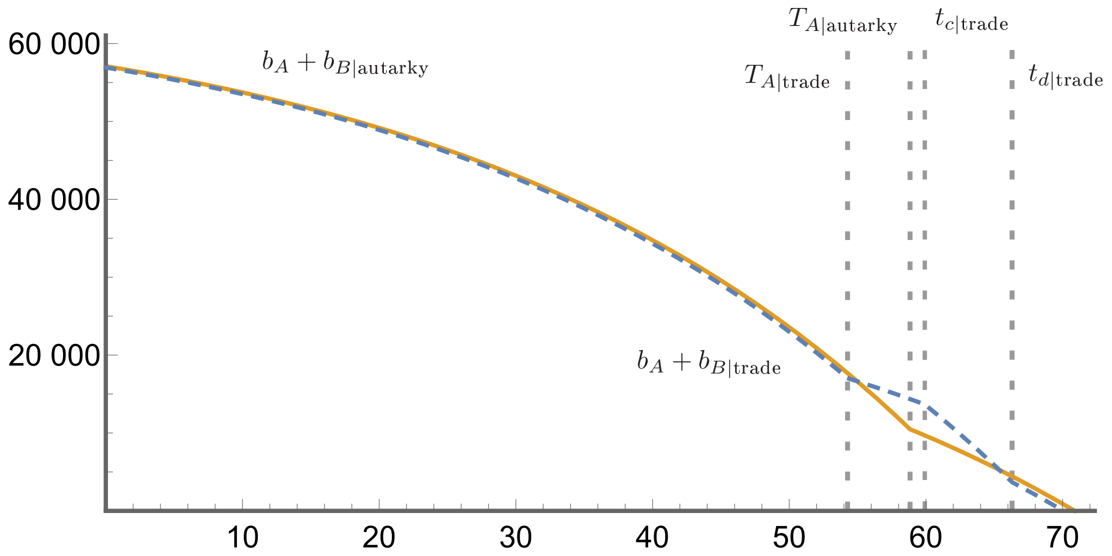


Figure 18: Fossil fuel extraction paths with strategic behavior and $\alpha = 0.8$

doxes. In an empirical calibration of a unilaterally regulated economy with oil and e-fuels it turns out that green trade flattens the extraction path both when the regulating country acts strategically and non-strategically. In a more hypothetical scenario with significant conversion losses, green trade may steepen the extraction path, accelerate global warming and be harmful for climate change.

Our analysis can be extended in various directions. First, one could replace the climate-damage function by a ceiling on carbon emissions. Second, it may be important in the future to check the robustness of our results when extraction costs are convex and stock-dependent. Third, one could introduce further policies, e.g. green energy subsidies or import tariffs. Fourth, one could assume that both countries suffer from climate damage, have climate policies such that a dynamic game is played between the two countries. Finally, one could use subgame-perfect strategies to analyze the strategic behavior of the coalition.

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A Appendix

A.1 Microfoundation

Let ℓ be a composite production factor, say labor and land, and the (inverse) production functions of country $i = A, B$ be given by

$$\ell_{xi}(t) = x_i^s(t), \tag{44}$$

$$\ell_{bi}(t) = C(b_i^s(t)), \tag{45}$$

$$\ell_{gi}(t) = M_i(g_i^s(t)), \tag{46}$$

$$\ell_{qi}(t) = Q(g_{ij}^s(t)), \tag{47}$$

where $\ell_{xi}(t)$ is the input in the consumer good production of country i, $\ell_{bi}(t)$ is the input in the black energy generation of country i, $\ell_{gi}(t)$ is the input in the green energy generation of country i, and $\ell_{qi}(t)$ is land or labor necessary for converting and transporting green energy from country i to country $j \neq i$. Because the composite production factor is immobile and the consumer good is mobile, the constraints for the production factor and the consumer good are

$$\ell_{xi}(t) + \ell_{bi}(t) + \ell_{gi}(t) + \ell_{qi}(t) = \bar{\ell}_i, \tag{48}$$

$$x_A^s(t) + x_B^s(t) = x_A(t) + x_B(t), \tag{49}$$

where $\bar{\ell}_i$ is country i's endowment of the composite production factor. Inserting (44)-(49) into (1) and accounting for the climate damage (6) yields the aggregate welfare

$$\sum_i [U(y_i(t)) + x_i(t) - H(E(t))]$$
$$= \sum_i \left[U(y_i(t)) + \bar{\ell}_i - C(b_i^s(t)) - M_i(g_i^s(t)) - Q(g_{ij}^s(t)) - H(E(t))\right]. \tag{50}$$

A.2 Laissez faire

A.2.1 Proof of Lemma 1

The representative green energy firm of country $i = A, B$ maximizes

$$L = p_i^g g_{ii}^s + \alpha p_j^g g_{ij}^s - M_i\left(g_{ii}^s + g_{ij}^s\right) - Q\left(g_{ij}^s\right) + \zeta_{ii} g_{ii}^s + \zeta_{ij} g_{ij}^s,$$

where ζ_{ii} and ζ_{ij} are the multipliers of the non-negativity conditions $g_{ii}^s \geq 0$ and $g_{ij}^s \geq 0$, with $i, j = A, B$ and $i \neq j$. The first-order conditions give

$$p_i^g = M_i'(g_{ii}^s + g_{ij}^s) - \zeta_{ii}, \tag{51}$$

$$\alpha p_j^g = M_i'(g_{ii}^s + g_{ij}^s) + Q'(g_{ij}^s) - \zeta_{ij}. \tag{52}$$

The complementary slackness conditions read

$$\zeta_{ii} \geq 0, \zeta_{ii} g_{ii}^s = 0, \quad \zeta_{ij} \geq 0, \zeta_{ij} g_{ij}^s = 0.$$

Define $\bar{p}_i$ as the energy price in country $i = A, B$ with only domestic green energy supply, i.e. $\bar{p}_i$ is given by $\bar{p}_i = U'\left(g_{ii}^s\right) = M_i'\left(g_{ii}^s\right)$. By assumption, the fuel firm's initial scarcity rent is such that the initial fuel price $p^b(0)$ undercuts $\bar{p}_B < \bar{p}_A$ implying that fuel is used in both countries and $p_A(t) = p_B(t) = p^b(t) = p_A^g(t) = p_B^g(t)$ in early periods. Consequently, (51) and (52) yield

$$\zeta_{ii} = -[1 - \alpha] p^b - Q'(g_{ij}^s) + \zeta_{ij}. \tag{53}$$

Suppose that g_{ij}^s is positive. Then, (53) implies $\zeta_{ii} < 0$, which violates the complementary slackness condition. The contradictions rules out green energy exports.

Suppose that $g^s_{AA} > 0$, $g^s_{BB} = 0$; $g^s_{AA} = 0$, $g^s_{BB} > 0$ or $g^s_{AA} = g^s_{BB} = 0$ holds. In the first case, (51) and (52) imply $p^b = M'_A(g^s_{AA}) > 0$ and $p^b = 0$. In the second case, we get $p^b = 0$ and $p^b = M'_B(g^s_{BB}) > 0$. In third case, we get $p^b = p^g_i = 0$. In the first two cases, the contradictions rules out $g^s_{BB} = 0$ and $g^s_{AA} = 0$, respectively. In the third case, a fuel producer price of zero is ruled out by a limited fuel stock. Thus, domestic green energy supply has to be positive as long as the energy prices are equal in the two countries.

Due to the Hotelling-rule (14), the scarcity rent and, therefore, the fossil fuel producer price p^b continuously increases in time. For $t < T_B$, $\bar{p}_B = U'\left(g^s_{BB}\right) = M'_B\left(g^s_{BB}\right) > p^b$ holds, while $\bar{p}_B = U'\left(g^s_{BB}\right) = M'_B\left(g^s_{BB}\right) \leq p^b$ holds for all $t \geq T_B$. Thus, the above arguments hold for all $t \in [0, T_B)$. $\square$

A.2.2 Proof of Proposition 1

Consider Phase I. The identity $p^b = p^{\text{LF}}_A = p^{\text{LF}}_B = c + \lambda$ and $g^{\text{LF}}_{AB} = g^{\text{LF}}_{BA} = 0$ are direct implications of Lemma 1. Due to the Hotelling-rule (14), $\dot{p}^{\text{LF}}_i = \rho\lambda > 0$ holds. Differentiating (51) with respect to time and taking $g^s_{AB} = g^s_{BA} = 0$ into account yields $\dot{g}^{\text{LF}}_{ii} = \frac{\rho\lambda}{M''_i} > 0$. Differentiating (20) with respect to time gives $\dot{b}^d_i + \dot{g}^d_i = D'\rho\lambda < 0$. In equilibrium, fossil fuel supply in country i equals demand, so that $\dot{b}^{\text{LF}}_i = D'\rho\lambda - \dot{g}^{\text{LF}}_{ii} < 0$.

Consider Phase II. Because $\bar{p}_A > c + \lambda > \bar{p}_B$, no fossil fuel is used in country B, while $b^{\text{LF}}_A > 0$. The latter implies $p^{\text{LF}}_A = c + \lambda$. By definition of Phase II, $\alpha p^{\text{LF}}_A - \bar{p}_B < Q'(0)$, so that no green energy is traded. Then, the energy market equilibrium in country B is not subject to any time-dependent variable implying $\dot{p}^{\text{LF}}_B = \dot{g}^{\text{LF}}_{BB} = 0$. In case of country A, the Hotelling-rule (14) and (51) imply $\dot{p}^{\text{LF}}_A = \rho\lambda > 0$ and $\dot{g}^{\text{LF}}_{AA} = \frac{\rho\lambda}{M''_A} > 0$. Differentiating (20) with respect to time yields $\dot{b}^{\text{LF}}_A = D'\rho\lambda - \dot{g}^{\text{LF}}_{AA} < 0$.

Consider Phase III. The phase only exists if $\bar{p}_A$ is sufficiently high, so that the difference $\alpha[c + \lambda(t)] - \bar{p}_B$ can exceed $Q'(0)$. This is ensured by sufficiently high marginal production costs of green energy in country A. Then, by definition of Phase III, $\alpha p^{\text{LF}}_A - \bar{p}_B > Q'(0)$ implying $g^{\text{LF}}_{BA} > 0$. In country A, the energy price is given by $p^{\text{LF}}_A = c + \lambda < U'(g^{\text{LF}}_{AA} + \alpha g^{\text{LF}}_{BA})$. In country B, (51) and (52) hold with $\zeta_{BB} = \zeta_{BA} = 0$ implying $p^{\text{LF}}_B = \alpha p^{\text{LF}}_A - Q'(g^{\text{LF}}_{BA})$. Due to the Hotelling-rule (14), $\dot{p}^{\text{LF}}_A = \rho\lambda > 0$. Differentiating (51) and (52) with respect to time and taking (20) into account yield

$$
\begin{aligned}
\frac{\dot{g}^{\text{LF}}_{BB}}{D'} &= M''_B\left[\dot{g}^{\text{LF}}_{BB} + \dot{g}^{\text{LF}}_{BA}\right], \\
\alpha\rho\lambda &= M''_B\left[\dot{g}^{\text{LF}}_{BB} + \dot{g}^{\text{LF}}_{BA}\right] + Q''\dot{g}^{\text{LF}}_{BA}.
\end{aligned}
$$

Solving gives

$$\dot{g}_{BB}^{\text{LF}} = \frac{D' M_B''}{Q'' \left[1 - D' M_B''\right] + M_B''} \alpha \rho \lambda < 0,$$
$$\dot{g}_{BA}^{\text{LF}} = \frac{1 - D' M_B''}{Q'' \left[1 - D' M_B''\right] + M_B''} \alpha \rho \lambda > 0.$$

From (20), we get $\dot{p}_B^{\text{LF}} = U'' \dot{g}_{BB}^{\text{LF}} > 0$ for the energy price in country B. For country A, the differentiation of (51) yields $\dot{g}_{AA}^{\text{LF}} = \frac{\rho\lambda}{M_A''} > 0$. Finally, the differentiation of (20) gives $\dot{b}_A^{\text{LF}} = D' \rho \lambda - \dot{g}_{AA}^{\text{LF}} - \alpha \dot{g}_{BA}^{\text{LF}} < 0$. Note that the price difference $\alpha p_A^{\text{LF}} - p_B^{\text{LF}}$ increases in time, because $\alpha \dot{p}_A^{\text{LF}} - \dot{p}_B^{\text{LF}} = Q'' \dot{g}_{BA}^{\text{LF}} > 0$.

Consider Phase IV. Fossil fuel extraction ends when $p_A^{\text{LF}}(t) = U'(g_{AA}^{\text{LF}}(t) + g_{BA}^{\text{LF}}(t)) \leq c + \lambda(t)$ holds the first time at $t = T$. Because the price difference $\alpha p_A^{\text{LF}}(t) - p_B^{\text{LF}}(t)$ increased until time T, green energy exports g_{BA}^{LF} are positive. Because fossil fuel extraction vanished, the energy market equilibria in both country A and B are not subject to any time-dependent variable implying constant prices and quantities. □

A.3 Social optimum

A.3.1 Solution of the social planner

The Lagrangian of the social planner reads

$$\begin{aligned}
L = {} & U(b_A^d + g_A^d) + \bar{\ell}_A - M_A(g_{AA}^s + g_{AB}^s) - Q(g_{AB}^s) - H(E) \\
& + U(b_B^d + g_B^d) + \bar{\ell}_B - M_B(g_{BB}^s + g_{BA}^s) - Q(g_{BA}^s) - H(E) - C(b_A^s + b_B^s) \\
& - \kappa \left[b_A^s + b_B^s\right] + \tilde{\theta} \left[b_A^d + b_B^d - \gamma E\right] + \mu \left[b_A^s + b_B^s - b_A^d - b_B^d\right] + \eta_A \left[g_{AA}^s + \alpha g_{BA}^s - g_A^d\right] \\
& + \eta_B \left[g_{BB}^s + \alpha g_{AB}^s - g_B^d\right] + \zeta_{b_A^s} b_A^s + \zeta_{b_B^s} b_B^s + \zeta_{b_A^d} b_A^d + \zeta_{b_B^d} b_B^d + \zeta_{g_A^d} g_A^d + \zeta_{g_B^d} g_B^d \\
& + \zeta_{g_{AA}^s} g_{AA}^s + \zeta_{g_{AB}^s} g_{AB}^s + \zeta_{g_{BB}^s} g_{BB}^s + \zeta_{g_{BA}^s} g_{BA}^s,
\end{aligned}$$

where κ is the costate of the fuel stock S, $\tilde{\theta}$ the costate of the emission stock, μ the multiplier of the constraint $b_A^s + b_B^s = b_A^d + b_B^s$, η_i the multiplier of the constraint $g_i^d = g_{ii}^s - g_{ji}^s$ and $\zeta_{b_A^s}$, $\zeta_{b_B^s}$, $\zeta_{b_A^d}$, $\zeta_{b_A^d}$, $\zeta_{b_B^d}$, $\zeta_{g_A^d}$, $\zeta_{g_B^d}$, $\zeta_{g_{AA}^s}$, $\zeta_{g_{AB}^s}$, $\zeta_{g_{BB}^s}$ and $\zeta_{g_{BA}^s}$ the multipliers of the non-negativity constraints $b_A^s, b_B^s, b_A^d, b_B^d, g_A^d, g_B^d, g_{AA}^s, g_{AB}^s, g_{BB}^s, g_{BA}^s \geq 0$. The first-order conditions with respect to fossil fuel supply give

$$C'(b_A^s + b_B^s) + \kappa - \zeta_{b_A^s} = \mu, \tag{54}$$
$$C'(b_A^s + b_B^s) + \kappa - \zeta_{b_B^s} = \mu, \tag{55}$$

with $\zeta_{b_A^s} \geq 0, \zeta_{b_A^s} b_A^s = 0$ and $\zeta_{b_B^s} \geq 0, \zeta_{b_B^s} b_B^s = 0$ as the related complementary slackness conditions.

Lemma 2 *Fossil fuel extraction is either nil in both countries or positive in both countries.*

Proof Suppose that $b_i^s > b_j^s = 0$. Then, (54) and (55) imply $C'(b_i^s) + \kappa = C'(b_i^s) + \kappa - \zeta_{b_j^s}$. The equation only holds for $\zeta_{b_j^s} = 0$ implying $b_j^s > 0$. □

The first-order conditions with respect to the stocks and the transversality conditions yield

$$\hat{\kappa} = \rho, \tag{56}$$

$$\dot{\tilde{\theta}} = \tilde{\theta}[\rho + \gamma] + 2H'(E), \tag{57}$$

$$(a)\ \lim_{t\to\infty} e^{-\rho t}\kappa(t)\left[S(t) - S^{opt}(t)\right] \geq 0 \qquad (b)\ \lim_{t\to\infty} e^{-\rho t}\tilde{\theta}(t)\left[E(t) - E^{opt}(t)\right] \geq 0. \tag{58}$$

Solving (57) yields $\tilde{\theta}(t) = \tilde{\theta}_0 e^{[\rho+\gamma]t} - \frac{2h}{\rho+\gamma}$, where $\tilde{\theta}_0$ is a constant of integration. Substituting into (58)(b) shows that $\tilde{\theta}_0 = 0$ to ensure the transversality condition, so that the negative costate of the emission stock equals the social costs of carbon, i.e.

$$\theta(t) := -\tilde{\theta}(t) = \frac{2h}{\rho + \gamma} \tag{59}$$

The first-order conditions with respect to energy demand yield

$$U'(b_A^d + g_A^d) = -\tilde{\theta}_A + \mu - \zeta_{b_A^d}, \tag{60}$$

$$U'(b_A^d + g_A^d) = \eta_A - \zeta_{g_A^d}, \tag{61}$$

$$U'(b_B^d + g_B^d) = -\tilde{\theta}_A + \mu - \zeta_{b_B^d}, \tag{62}$$

$$U'(b_B^d + g_B^d) = \eta_B - \zeta_{g_B^d}, \tag{63}$$

with $\zeta_{b_A^d} \geq 0, \zeta_{b_A^d} b_A^d = 0$, $\zeta_{b_B^d} \geq 0, \zeta_{b_B^d} b_B^d = 0$, $\zeta_{g_A^d} \geq 0, \zeta_{g_A^d} g_A^d = 0$ and $\zeta_{g_B^d} \geq 0, \zeta_{g_B^d} g_B^d = 0$ as the related complementary slackness conditions. With respect to green energy supply and export, we get

$$\eta_A = M_A'(g_{AA}^s + g_{AB}^s) - \zeta_{g_{AA}^s}, \tag{64}$$

$$\alpha\eta_B = M_A'(g_{AA}^s + g_{AB}^s) + Q'(g_{AB}^s) - \zeta_{g_{AB}^s}, \tag{65}$$

$$\eta_B = M_B'(g_{BB}^s + g_{BA}^s) - \zeta_{g_{BB}^s}, \tag{66}$$

$$\alpha\eta_A = M_B'(g_{BB}^s + g_{BA}^s) + Q'(g_{BA}^s) - \zeta_{g_{BA}^s}, \tag{67}$$

with $\zeta_{g_{AA}^s} \geq 0, \zeta_{g_{AA}^s} g_{AA}^s = 0$, $\zeta_{g_{AB}^s} \geq 0, \zeta_{g_{AB}^s} g_{AB}^s = 0$, $\zeta_{g_{BB}^s} \geq 0, \zeta_{g_{BB}^s} g_{BB}^s = 0$ and $\zeta_{g_{BA}^s} \geq$

$0, \zeta_{g^s_{BA}} g^s_{BA} = 0$. By substituting (54), (55), (59) and (64) - (67) into (60) - (63), we get

$$\begin{aligned}
U'(b^d_A + g^d_A) &= C'(b^s_A + b^s_B) + \kappa + \theta - \zeta_{b^s_A} - \zeta_{b^d_A} \\
&= M'_A(g^s_{AA} + g^s_{AB}) - \zeta_{g^s_{AA}} - \zeta_{g^d_A} \\
&= \frac{M'_B(g^s_{BB} + g^s_{BA})}{\alpha} + \frac{Q'(g^s_{BA})}{\alpha} - \frac{\zeta_{g^s_{BA}}}{\alpha} - \zeta_{g^d_A}, \\
U'(b^d_B + g^d_B) &= C'(b^s_A + b^s_B) + \kappa + \theta - \zeta_{b^s_B} - \zeta_{b^d_B} \\
&= M'_B(g^s_{BB} + g^s_{BA}) - \zeta_{g^s_{BB}} - \zeta_{g^d_B} \\
&= \frac{M'_A(g^s_{AA} + g^s_{AB})}{\alpha} + \frac{Q'(g^s_{AB})}{\alpha} - \frac{\zeta_{g^s_{AB}}}{\alpha} - \zeta_{g^d_B}.
\end{aligned}$$

A.3.2 Proof of Proposition 3

Consider Phase I. By assumption, the fuel endowment is sufficiently large such that $\kappa(0) + c$ falls short of $U'(\bar{g}_{BB}) < U'(\bar{g}_{AA})$ implying the use of fossil fuel in both countries. Consequently, fuel is extracted in both countries due to lemma 2. Thus, $\zeta_{b^s_i} = \zeta_{b^d_i} = 0$, so that the first lines of (25) and (26) yield

$$U'(y_A) = C'(b^s_A + b^s_B) + \kappa + \theta = U'(y_B).$$

Suppose that green energy is exported from country i to country j implying $\zeta_{g^d_j} = \zeta_{g^s_{ij}} = 0$. Then, (25) and (26) give

$$\begin{aligned}
U'(y_i) = C'(b^s_A + b^s_B) + \kappa + \theta = \frac{M'_i(g^s_i)}{\alpha} + \frac{Q'(g^s_{ij})}{\alpha} \\
\Leftrightarrow Q'(g^s_{ij}) = -[1-\alpha]\left[C'(b^s_A + b^s_B) + \kappa + \theta\right] - \zeta_{g^s_{ii}} - \zeta_{g^d_i} < 0.
\end{aligned}$$

Because the marginal transportation costs are non-negative, energy exports are ruled out. Domestic green energy production and consumption is profitable, because $M'_i(0) < \kappa(0) + c$ implying $\zeta_{g^d_i} = \zeta_{g^s_{ii}} = 0$, so that

$$U'(b^{\mathrm{E}}_i + g^{\mathrm{E}}_{ii}) = C'(b^{\mathrm{E}}_A + b^{\mathrm{E}}_B) + \kappa + \theta = M'_i(g^{\mathrm{E}}_{ii}).$$

Differentiating with respect to time and solving yield $\dot{g}^{\mathrm{E}}_{ii} = \frac{\rho\kappa}{M''_i} > 0$ and $\dot{b}^{\mathrm{E}}_i = \frac{D'M''_i - 1}{M''_i}\rho\kappa < 0$.

Consider Phase II. By definition, $C'(0) + \kappa + \theta < M'_A(g^s_A)$, so that fossil fuel is used in country A and lemma 2 implies $\zeta_{b^s_A} = \zeta_{b^s_B} = \zeta_{b^s} = \zeta_{b^d_A} = 0$. In contrast, the first line of (26) only holds for $\zeta_{b^d_B} > 0$ implying $b^d_B = 0$. Suppose that green energy is exported from country A to B, so that $\zeta_{g^s_{AB}} = \zeta_{g^d_B} = 0$. Then, (25) and (26) give

$$\begin{aligned}
U'(y_B) = C'(b^s_A + b^s_B) + \kappa + \theta - \zeta_{b^d_B} = \frac{M'_A(g^s_A)}{\alpha} + \frac{Q'(g^s_{AB})}{\alpha} \\
\Leftrightarrow Q'(g^s_{AB}) = -[1-\alpha]\left[C'(b^s_A + b^s_B) + \kappa + \theta\right] - \alpha\zeta_{b^d_B} - \zeta_{g^s_{AA}} - \zeta_{g^d_A} < 0.
\end{aligned}$$

The contradiction implies $g^s_{AB} = 0$. Without green energy imports and without fossil fuels, domestic green energy production must be used in country B, because $\lim_{y_B \to 0} U'(y_B) = \infty$. Consequently, $y_B = g^d_B = g^s_{BB} > 0$ and $\zeta_{g^s_{BB}} = \zeta_{g^d_B} = 0$. The differentiation of $U'(g^{\mathrm{E}}_{BB}) = M'_B(g^{\mathrm{E}}_{BB})$ with respect to time yields $\dot{g}^{\mathrm{E}}_{BB} = 0$.
In country A, $C'(0)+\kappa+\theta > M'_A(0)$ implies that green energy production and consumption is profitable implying $\zeta_{g^d_A} = \zeta_{g^s_{AA}} = 0$. Substituting into (25) and (26) and solving give

$$\alpha U'(y_A) - U'(y_B) = Q'(g^s_{BA}) - \zeta_{g^s_{BA}}. \tag{68}$$

By definition of Phase II, the weighted marginal utility difference falls short of $Q'(0)$, so that $\zeta_{g^s_{BA}} > 0$ and $g^s_{BA} = 0$. Differentiating $U'(b^{\mathrm{E}}_A + g^{\mathrm{E}}_{AA}) = C'(b^{\mathrm{E}}_A + b^{\mathrm{E}}_B) + \kappa + \theta = M'_A(g^{\mathrm{E}}_{AA})$ with respect to time and solving give $\dot{g}^{\mathrm{E}}_{AA} = \frac{\rho\kappa}{M''_A} > 0$ and $\dot{b}^{\mathrm{E}}_A = \frac{D'M''_A - 1}{M''_A}\rho\kappa < 0$.

Consider Phase III. The arguments for Phase II made with respect to fossil fuel use, exports g^s_{AB}, green energy production g^s_{BB} and g^s_{AA} hold in an analogous way. However, by definition, the difference of (68) is now larger than $Q'(0)$ implying energy exports from country B to country A. Then, (25) and (26) read

$$U'(b^{\mathrm{E}}_A + g^{\mathrm{E}}_{AA} + g^{\mathrm{E}}_{BA}) = C'(b^s_A + b^s_B) + \kappa + \theta = M'_A(g^{\mathrm{E}}_{AA}) = \frac{M'_B(g^{\mathrm{E}}_{BB} + g^{\mathrm{E}}_{BA})}{\alpha} + \frac{Q'(g^{\mathrm{E}}_{BA})}{\alpha},$$
$$U'(g^{\mathrm{E}}_{BB}) = M'_B(g^{\mathrm{E}}_{BB} + g^{\mathrm{E}}_{BA}).$$

Differentiating with respect to time and solving yield

$$\begin{aligned}
\dot{g}^{\mathrm{E}}_{AA} &= \frac{\rho\kappa}{M''_A} > 0,\\
\dot{g}^{\mathrm{E}}_{BA} &= \frac{1 - D'M''_B}{Q''[1 - D'M''_B] + M''_B}\alpha\rho\kappa > 0,\\
\dot{g}^{\mathrm{E}}_{BB} &= \frac{D'M''_B}{Q''[1 - D'M''_B] + M''_B}\alpha\rho\kappa < 0,\\
\dot{b}^{\mathrm{E}}_A &= \rho\kappa\left[-\frac{1 - D'M''_A}{M''_A} - \alpha\frac{1 - D'M''_B}{Q''[1 - D'M''_B] + M''_B}\right] < 0.
\end{aligned}$$

Consider Phase IV. By definition, fossil fuel extraction is nil in this phase, so that the transversality condition (58)(a) implies the exhaustion of the fuel stock at time T_A, so that the first lines of (25) and (26) can be ignored. During Phase III, the utility difference $\alpha U'(y_A) - U'(y_B)$ increased, because of of $\dot{g}^{\mathrm{E}}_{BA} > 0$. Consequently, at time T (68) holds for $\zeta^s_{BA} = 0$ implying energy exports from country B to country A. Without fossil fuels and, therefore, without the scarcity rent κ, the allocation is not subject to any time-dependent variable implying constant quantities. □

A.4 Fuel tax

A.4.1 The optimal fuel tax of a non-strategic government

The Lagrangian of country A's government reads

$$L = U(b_A^d + g_A^d) + \bar{\ell}_A + \Pi_A + s_A\Pi_F - p_A^n b_A^d - p_A g_A^d - H(E) + \tilde{\theta}_A\left[b_A^d + b_B^d - \gamma E\right] + \tilde{\zeta}_{b_A^d} b_A^d,$$

where p_A^n is the energy price net of taxes, $\tilde{\theta}_A$ is costate of the emission stock and $\tilde{\zeta}_{b_A^d}$ the multiplier of the non-negativity constraint $b_A^d \geq 0$. The first-order conditions and the transversality condition yield

$$U'(b_A^d + g_A^d) = p_A^n - \tilde{\theta}_A - \tilde{\zeta}_{b_A^d}, \tag{69}$$

$$\dot{\tilde{\theta}}_A = [\rho + \gamma]\tilde{\theta}_A + H', \tag{70}$$

$$\lim_{t\to\infty} e^{-\rho t}\tilde{\theta}(t)\left[E(t) - E^{opt}(t)\right] \geq 0. \tag{71}$$

Solving (70) gives $\tilde{\theta}_A(t) = \tilde{\theta}_{A0}e^{[\rho+\gamma]t} - \frac{h}{\rho+\gamma}$. Because the transversality condition (71) would be violated for $\tilde{\theta}_{A0} \neq 0$, we get $\tilde{\theta}_A(t) = -\frac{h}{\rho+\gamma}$. The first-order condition for the utility maximum of the individual is identical to the laissez-faire case and, therefore, given by

$$U'(b_A^d + g_A^d) = p_A^n + \tau - \zeta_{b_A^d}. \tag{72}$$

Assuming $\zeta_{b_A^d} = \tilde{\zeta}_{b_A^d}$, the identity of (69) and (72) requires

$$\tau^{\mathrm{U}}(t) = -\tilde{\theta}_A(t) = \frac{h}{\rho+\gamma}. \tag{73}$$

A.4.2 Proof of propositions 4

Suppose $t \in [0, T_A)$. With green energy trade, we get $p_A(t) = c + \lambda(t) + \tau^{\mathrm{U}}$ and $p_B(t) = c + \lambda$ implying

$$g_{AA}^{\mathrm{U}}(t) = \frac{c + \lambda(t) + \tau^{\mathrm{U}}}{m_A}, \tag{74}$$

$$g_{BA}^{\mathrm{U}}(t) = \frac{\alpha[c + \lambda(t) + \tau^{\mathrm{U}}]}{m_B + q}, \tag{75}$$

$$b_A^{\mathrm{U}}(t) = D(c + \lambda(t) + \tau^{\mathrm{U}}) - g_{AA}^{\mathrm{U}}(t) - \alpha g_{BA}^{\mathrm{U}}(t), \tag{76}$$

$$b_B^{\mathrm{U}}(t) = D(c + \lambda(t)). \tag{77}$$

Ceteris paribus, the energy prices in country A and B without green energy trade are given by $p_A(t) = c + \lambda(t) + \tau^{\text{U}}$ and $p_B(t) = c + \lambda$ implying

$$g_{AA}^{\text{AU}}(t) = \frac{c + \lambda(t) + \tau^{\text{U}}}{m_A}, \tag{78}$$

$$g_{BB}^{\text{AU}}(t) = \frac{c + \lambda(t)}{m_B}, \tag{79}$$

$$b_A^{\text{AU}}(t) = D(c + \lambda(t) + \tau^{\text{U}}) - g_{AA}^{\text{AU}}(t), \tag{80}$$

$$b_B^{\text{AU}}(t) = D(c + \lambda(t)) - g_{BB}^{\text{AU}}(t). \tag{81}$$

Green energy trade increases the consumption of green energy if $g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t) > g_{AA}^{\text{AU}}(t) + g_{BB}^{\text{AU}}(t)$, which gives

$$\alpha g_{BA}^{\text{U}}(t) > g_{BB}^{\text{AU}}(t) \;\Leftrightarrow\; \alpha^2 m_B \tau^{\text{U}} > \left[q + m_B - \alpha^2 m_B\right][c + \lambda(t)]\,. \tag{82}$$

The inequality holds, if both α and τ^{U} are sufficiently large. With respect to fossil fuel consumption, we get

$$[b_A^{\text{AU}}(t) + b_B^{\text{AU}}(t)] - [b_A^{\text{U}}(t) + b_B^{\text{U}}(t)] = \alpha g_{BA}^{\text{U}}(t) - g_{BB}^{\text{AU}}(t). \tag{83}$$

Thus, fossil fuel consumption is higher in case of green energy trade if (82) does not hold.

Suppose $t \in [T_A, t_c)$. With green energy trade, we get $p_A(t) = U'(g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t))$ and $p_B(t) = c + \lambda$ implying

$$g_{AA}^{\text{U}}(t) = \frac{p_A^{\text{U}}(t)}{m_A} = \frac{a[m_B + q]}{\alpha^2 m_A + [m_B + q][1 + z m_A]}, \tag{84}$$

$$g_{BA}^{\text{U}}(t) = \frac{\alpha a m_A}{\alpha^2 m_A + [m_B + q][1 + z m_A]}, \tag{85}$$

$$b_B^{\text{U}}(t) = D(c + \lambda(t)). \tag{86}$$

Ceteris paribus, the energy prices in country A and B without green energy trade are given by $p_A(t) = c + \lambda(t) + \tau^{\text{U}}$ and $p_B(t) = c + \lambda$ if fuel is used in both countries. Then, (78) - (81) hold. Green energy trade increases the consumption of green energy if $g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t) > g_{AA}^{\text{AU}}(t) + g_{BB}^{\text{AU}}(t)$, which gives

$$\frac{\alpha^2 a m_A + a[m_B + q]}{\alpha^2 m_A + [m_B + q][1 + z m_A]} > \frac{c + \lambda + \tau^{\text{U}}}{m_A} + \frac{c + \lambda}{m_B} \tag{87}$$

Because $p_A^{\text{U}}(t) < c + \lambda(t) + \tau^{\text{U}}$, the inequality does not hold if α is sufficiently low or τ^{U} is sufficiently high. With respect to fossil fuel use, we get

$$b_A^{\text{AU}}(t) + b_B^{\text{AU}}(t) - b_B^{\text{U}}(t) = b_A^{\text{AU}}(t) - g_{BB}^{\text{AU}}(t). \tag{88}$$

Because $b_A^{\text{AU}}(t)$ decreases in c, $\lambda(t)$ and τ, while $g_{BB}^{\text{AU}}(t)$ increases in c and $\lambda(t)$, fuel use is higher under green autarky, if τ^{U} is sufficiently low, i.e. $\tau^{\text{U}} < \frac{am_A m_B - [c+\lambda(t)][m_A+m_B+zm_A m_B]}{m_B[1+zm_A]}$ has to hold.

Consider the case that no fuel is used in country A without green energy trade. Then, the energy prices in country A and B without green energy trade are given by $p_A^{\text{AU}}(t) = U'(g_{AA}^{\text{AU}}(t))$ and $p_B(t) = c + \lambda$ implying (79), (81) and

$$g_{AA}^{\text{AU}}(t) = \frac{p_A^{\text{AU}}(t)}{m_A} = \frac{a}{1+zm_A}. \tag{89}$$

Green energy trade increases the consumption of green energy if $g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t) > g_{AA}^{\text{AU}}(t) + g_{BB}^{\text{AU}}(t)$. Because $g_{AA}^{\text{U}}(t) < g_{AA}^{\text{AU}}(t)$, the inequality does not hold if α is sufficiently low. With respect to fossil fuel use, we get

$$b_B^{\text{AU}}(t) - b_B^{\text{U}}(t) = -g_{BB}^{\text{AU}}(t) < 0. \tag{90}$$

Thus, fuel use is higher with green energy trade.

Suppose $t \in [t_c, T_B)$. With green energy trade, we get $p_A(t) = U'(g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t))$ and $p_B(t) = c + \lambda$ implying

$$g_{AA}^{\text{U}}(t) = \frac{p_A^{\text{U}}(t)}{m_A} = \frac{aq + \alpha[c+\lambda(t)]}{\alpha^2 m_A + q + qzm_A}, \tag{91}$$

$$g_{BA}^{\text{U}}(t) = \frac{\alpha a m_A - [1+zm_A][c+\lambda(t)]}{\alpha^2 m_A + q + qzm_A]}, \tag{92}$$

$$g_{BB}^{\text{U}}(t) = \frac{c+\lambda(t)}{m_B} - g_{BA}^{\text{U}}(t), \tag{93}$$

$$b_B^{\text{U}}(t) = D(c+\lambda(t)) - g_{BB}^{\text{U}}(t). \tag{94}$$

Ceteris paribus, the energy prices in country A and B without green energy trade are given by $p_A(t) = c + \lambda(t) + \tau^{\text{U}}$ and $p_B(t) = c + \lambda$ if fuel is used in both countries. Then, (78) - (81) hold. Green energy trade increases the consumption of green energy if $g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t) + g_{BB}^{\text{U}}(t) > g_{AA}^{\text{AU}}(t) + g_{BB}^{\text{AU}}(t)$, which gives

$$\frac{p_A^{\text{U}}(t)}{m_A} - \frac{p_A^{\text{AU}}(t)}{m_A} - [1-\alpha] g_{BA}^{\text{U}}(t) > 0. \tag{95}$$

Because $p_A^{\text{U}}(t) < p_A^{\text{AU}}(t)$, the inequality does not hold, i.e. green energy trade lowers green energy consumption ceteris paribus. With respect to fossil fuel use, we get

$$b_A^{\text{AU}}(t) + b_B^{\text{AU}}(t) - b_B^{\text{U}}(t) = b_A^{\text{AU}}(t) - g_{BA}^{\text{U}}(t). \tag{96}$$

Because $b_A^{\text{AU}}(t)$ decreases in τ, the expression is positive if τ is sufficiently low, i.e. if $\tau^{\text{U}} < \frac{am_A}{1+zm_A} - \frac{\alpha a m_A^2}{[1+zm_A][\alpha^2 m_A + q + qzm_A]} - \frac{q[1+zm_A] - m_A[1-\alpha^2]}{\alpha^2 m_A + q + qzm_A}[c + \lambda(t)]$.

Consider the case that no fuel is used in country A without green energy trade. Then, the energy prices in country A and B without green energy trade are given by $p_A^{\text{AU}}(t) = U'(g_{AA}^{\text{AU}}(t))$ and $p_B(t) = c+\lambda$ implying (79), (81) and (89). Green energy trade increases the consumption of green energy if $g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t) + g_{BB}^{\text{U}}(t) > g_{AA}^{\text{AU}}(t) + g_{BB}^{\text{AU}}(t)$, which yields (95). That is, green energy trade lowers green energy consumption ceteris paribus. With respect to fossil fuel use, we get

$$b_B^{\text{AU}}(t) - b_B^{\text{U}}(t) = -g_{BA}^{\text{U}}(t) < 0. \tag{97}$$

Thus, green energy trade increases fossil fuel use ceteris paribus. □

A.4.3 Proof of proposition 6

Suppose $t \in [0, t_c)$. The argument made in A.4.2 with respect to $t \in [0, T_A)$ holds.

Suppose $t \in [t_c, t_d)$. With green energy trade, we get $p_A(t) = c + \lambda(t) + \tau^{\text{U}}$ and $p_B(t) = c + \lambda$ implying

$$g_{AA}^{\text{U}}(t) = \frac{c + \lambda(t) + \tau^{\text{U}}}{m_A}, \tag{98}$$

$$b_A^{\text{U}}(t) = D(c + \lambda(t) + \tau^{\text{U}}) - g_{AA}^{\text{U}}(t) - \alpha g_{BA}^{\text{U}}(t), \tag{99}$$

$$g_{BA}^{\text{U}}(t) = \frac{\alpha[c + \lambda(t) + \tau^{\text{U}}]}{q} - \frac{c + \lambda(t)}{q}, \tag{100}$$

$$g_{BB}^{\text{U}}(t) = \frac{c + \lambda(t)}{m_B} - \left\{\frac{\alpha[c + \lambda(t) + \tau^{\text{U}}]}{q} - \frac{c + \lambda(t)}{q}\right\}, \tag{101}$$

$$b_b^{\text{U}}(t) = D(c + \lambda(t)) - g_{BB}^{\text{U}}(t). \tag{102}$$

Ceteris paribus, the energy prices in country A and B without green energy trade are given by $p_A(t) = c + \lambda(t) + \tau^{\text{U}}$ and $p_B(t) = c + \lambda$ implying

$$g_{AA}^{\text{AU}}(t) = \frac{c + \lambda(t) + \tau^{\text{U}}}{m_A}, \tag{103}$$

$$g_{BB}^{\text{AU}}(t) = \frac{c + \lambda(t)}{m_B}, \tag{104}$$

$$b_A^{\text{AU}}(t) = D(c + \lambda(t) + \tau^{\text{U}}) - g_{AA}^{\text{AU}}(t), \tag{105}$$

$$b_B^{\text{AU}}(t) = D(c + \lambda(t)) - g_{BB}^{\text{AU}}(t) \tag{106}$$

Green energy trade increases the consumption of green energy if $g_{AA}^{\text{U}}(t) + \alpha g_{BA}^{\text{U}}(t) + g_{BB}^{\text{U}}(t) > g_{AA}^{\text{AU}}(t) + g_{BB}^{\text{AU}}(t)$, which gives

$$-[1-\alpha]\left\{\frac{\alpha[c + \lambda(t) + \tau^{\text{U}}]}{q} - \frac{c + \lambda(t)}{q}\right\} = -[1-\alpha] g_{BA}^{\text{U}}(t) > 0. \tag{107}$$

Because $g^{\text{U}}_{BA}(t) \geq 0$, the contradiction proves that less green energy is consumed with green energy trade than under green autarky. With respect to fossil fuel consumption, we get

$$[b^{\text{AU}}_A(t) + b^{\text{AU}}_B(t)] - [b^{\text{U}}_A(t) + b^{\text{U}}_B(t)] = g^{\text{U}}_{AA}(t) + \alpha g^{\text{U}}_{BA}(t) + g^{\text{U}}_{BB}(t) - g^{\text{AU}}_{AA}(t) - g^{\text{AU}}_{BB}(t) < 0. \quad (108)$$

Thus, fossil fuel consumption is higher in case of green energy trade.

Consider $t \in [t_d, T_A)$. Because green energy trade ends at time $t = t_d$, both green energy consumption and fossil fuel consumption are identical with and without the green energy trade option. □

A.5 Strategic policy

The Lagrangian of country A's government reads

$$\begin{aligned} L = {} & U(b^d_A + g^d_A) + \bar{\ell}_A + \alpha\left[p_B g^s_{AB} - p_A g^s_{BA}\right] - M_A(g^s_{AA} + g^s_{AB}) - Q(g^s_{AB}) \\ & + s_A\lambda\left[b^d_A + b^d_B\right] - [c+\lambda]b^d_A - hE + \tilde{\theta}_A\left[b^d_A + b^d_B - \gamma E\right] + \mu\rho\lambda \\ & - \eta\left[b^d_A + b^d_B\right] + \tilde{\zeta}_{b^d_A} b^d_A, \end{aligned} \quad (109)$$

where $p_A = c+\lambda+\tau$ and $p_B = c+\lambda$ hold if fuel is used in country A and B, respectively, such that the quantities g^d_A, g^s_{AA}, g^s_{AB}, g^s_{BA}, and b^d_B depend on λ. Otherwise prices and quantities do not depend on λ. The costates of the emission stock E, the fuel firm's scarcity rent λ and the fuel stock S are given by $\tilde{\theta}_A$, μ and η. $\tilde{\zeta}_{b^d_A}$ denotes the multiplier of the non-negativity condition $b^d_A \geq 0$. The first-order conditions yield

$$U' = c + \lambda - \tilde{\theta}_A - s_A\lambda + \eta - \tilde{\zeta}_{b^d_A}, \quad (110)$$

$$\dot{\mu} = \alpha\left[\frac{\partial p_A}{\partial\lambda}g^s_{BA} - \frac{\partial p_B}{\partial\lambda}g^s_{AB}\right] - s_A\left[b^d_A + b^d_B\right] + b^d_A + \left[\eta - s_A\lambda - \tilde{\theta}_A\right]\frac{\partial b^d_B}{\partial\lambda}, \quad (111)$$

$$\dot{\tilde{\theta}}_A = [\rho+\gamma]\,\tilde{\theta}_A + h, \quad (112)$$

$$\dot{\eta} = \rho\eta, \quad (113)$$

where $\frac{\partial p_A}{\partial\lambda} = 1$ for $t < T_A$, $\frac{\partial p_A}{\partial\lambda} \geq 0$ for $T_A \leq t < T_B$, $\frac{\partial p_A}{\partial\lambda} = 0$ for $t \geq T$, and $\frac{\partial p_B}{\partial\lambda} = 1$ for $t < T_B$, $\frac{\partial p_B}{\partial\lambda} \geq 0$ for $T_B \leq t < T_A$, $\frac{\partial p_B}{\partial\lambda} = 0$ for $t \geq T$.

The signs of $\frac{\partial p_A}{\partial\lambda}$ for $T_A \leq t < T_B$ and $\frac{\partial p_B}{\partial\lambda}$ for $T_B \leq t < T_A$ follow from lemma 3.

Lemma 3 *Suppose that country A applies a strategic climate policy. If $T_A < T_B$, $\frac{\partial p_A}{\partial\lambda} \geq 0$ holds for $t \in [T_A, T_B)$. If $T_B < T_A$, $\frac{\partial p_B}{\partial\lambda} \geq 0$ holds for $t \in [T_B, T_A)$.*

Proof Consider $\frac{\partial p_A}{\partial\lambda}$ for $T_A \leq t < T_B$. Because fuel consumption has ceased in country A but not in country B and $m_A > m_B$, $p_A(t) > p_B(t)$ holds ruling out green energy exports from country A to country B and implying $g^s_{AA}(t) > 0$. Thus, three cases may emerge:

(i) Country B exports a fraction of $g_B^s(t)$, so that $g_{BB}^s(t) > 0$, $g_{BA}^s(t) > 0$.

(ii) Country B exports $g_B^s(t)$ completely, so that $g_{BB}^s(t) = 0$, $g_{BA}^s(t) > 0$.

(iii) Country B does not export green energy, so that $g_{BB}^s(t) > 0$, $g_{BA}^s(t) = 0$.

In case (i), the energy market equilibrium is given by

$$b_B^{\text{ST}}(t) = D(c + \lambda(t)) - g_{BB}^{\text{ST}}(t), \tag{114}$$

$$g_{AA}^{\text{ST}}(t) + \alpha g_{BA}^{\text{ST}}(t) = D(p_A^{\text{ST}}(t)), \tag{115}$$

$$g_{AA}^{\text{ST}}(t) = \frac{p_A^{\text{ST}}(t)}{m_A}, \tag{116}$$

$$g_{BA}^{\text{ST}}(t) = \frac{\alpha p_A^{\text{ST}}(t) - [c + \lambda(t)]}{q}, \tag{117}$$

$$g_{BB}^{\text{ST}}(t) = \frac{c + \lambda(t)}{m_B} - \frac{\alpha p_A^{\text{ST}}(t) - [c + \lambda(t)]}{q}. \tag{118}$$

By differentiating (115) - (117) with respect to $\lambda(t)$, we get $\frac{\partial p_A^{\text{ST}}}{\partial \lambda} = \frac{\alpha m_A}{\alpha^2 m_A + q[1 - m_A D']} > 0$.

In case (ii), the energy market equilibrium is given by

$$g_{AA}^{\text{ST}}(t) = \frac{p_A^{\text{ST}}(t)}{m_A}, \tag{119}$$

$$g_{AA}^{\text{ST}}(t) + \alpha g_{BA}^{\text{ST}}(t) = D(p_A^{\text{ST}}(t)), \tag{120}$$

$$g_{BA}^{\text{ST}}(t) = \frac{\alpha p_A^{\text{ST}}(t)}{m_B + q}, \tag{121}$$

$$b_B^{\text{ST}}(t) = D(c + \lambda(t)). \tag{122}$$

By differentiating (119) - (121), we get $\frac{\partial p_A^{\text{ST}}}{\partial \lambda} = 0$.

In case (iii), the energy market equilibrium is given by

$$g_{AA}^{\text{ST}}(t) = \frac{p_A^{\text{ST}}(t)}{m_A}, \tag{123}$$

$$g_{AA}^{\text{ST}}(t) = D(p_A^{\text{ST}}(t)), \tag{124}$$

$$g_{BB}^{\text{ST}}(t) = \frac{c + \lambda(t)}{m_B}, \tag{125}$$

$$b_B^{\text{ST}}(t) = D(c + \lambda(t)) - g_{BB}^{\text{ST}}(t). \tag{126}$$

Differentiating (123) and (124) yields $\frac{\partial p_A^{\text{ST}}}{\partial \lambda} = 0$.

Consider $\frac{\partial p_B}{\partial \lambda}$ for $T_B \leq t < T_A$. Four cases may emerge:

(i) Country B exports a fraction of $g_B^s(t)$, so that $g_{BB}^s(t) > 0$, $g_{BA}^s(t) > 0$.

(ii) There is no green energy trade, so that $g_{BA}^s(t) = g_{AB}^s(t) = 0$.

(iii) Country A exports a fraction of $g_A^s(t)$, so that $g_{AA}^s(t) > 0$, $g_{AB}^s(t) > 0$.

(iv) Country A exports $g_A^s(t)$ completely, so that $g_{AA}^s(t) = 0$, $g_{AB}^s(t) > 0$.

In case (i), the energy market equilibrium is given by

$$b_A^{\text{ST}}(t) = D(c + \lambda + \tau) - g_{AA}^{\text{ST}}(t) - \alpha g_{BA}^{\text{ST}}(t), \tag{127}$$

$$g_{AA}^{\text{ST}}(t) = \frac{c + \lambda + \tau}{m_A}, \tag{128}$$

$$g_{BA}^{\text{ST}}(t) = \frac{\alpha[c + \lambda + \tau] - p_B^{\text{ST}}(t)}{q}, \tag{129}$$

$$g_{BB}^{\text{ST}}(t) + g_{BA}^{\text{ST}}(t) = \frac{p_B^{\text{ST}}(t)}{m_B}, \tag{130}$$

$$g_{BB}^{\text{ST}}(t) = D(p_B^{\text{ST}}(t)). \tag{131}$$

Differentiating (129) - (131) yields $\frac{\partial p_B^{\text{ST}}}{\partial \lambda} = \frac{\alpha m_B}{m_B + q[1 - m_B D']} > 0$.

In case (ii), the energy market equilibrium is given by

$$b_A^{\text{ST}}(t) = D(c + \lambda + \tau) - g_{AA}^{\text{ST}}(t), \tag{132}$$

$$g_{AA}^{\text{ST}}(t) = \frac{c + \lambda + \tau}{m_A}, \tag{133}$$

$$g_{BB}^{\text{ST}}(t) = D(p_B^{\text{ST}}(t)), \tag{134}$$

$$g_{BB}^{\text{ST}}(t) = \frac{p_B^{\text{ST}}(t)}{m_B}. \tag{135}$$

By differentiating the last two equations, we get $\frac{\partial p_B^{\text{ST}}}{\partial \lambda} = 0$.

In case (iii), the energy market equilibrium is given by

$$b_A^{\text{ST}}(t) = D(c + \lambda + \tau) - g_{AA}^{\text{ST}}(t), \tag{136}$$

$$g_{BB}^{\text{ST}}(t) + \alpha g_{AB}^{\text{ST}}(t) = D(p_B^{\text{ST}}(t)), \tag{137}$$

$$g_{BB}^{\text{ST}}(t) = \frac{p_B^{\text{ST}}(t)}{m_B}, \tag{138}$$

$$g_{AB}^{\text{ST}}(t) = \frac{\alpha p_B^{\text{ST}}(t) - [c + \lambda + \tau]}{q}, \tag{139}$$

$$g_{AA}^{\text{ST}}(t) = \frac{c + \lambda + \tau}{m_A} - \frac{\alpha p_B^{\text{ST}}(t) - [c + \lambda + \tau]}{q}. \tag{140}$$

By differentiating (137) - (139), we get $\frac{\partial p_B^{\text{ST}}}{\partial \lambda} = \frac{\alpha m_B}{\alpha^2 m_B + q[1 - m_B D']} > 0$.

In case (iv), the energy market equilibrium is given by

$$g_{BB}^{\mathrm{ST}}(t) = \frac{p_B^{\mathrm{ST}}(t)}{m_B}, \tag{141}$$

$$g_{BB}^{\mathrm{ST}}(t) + \alpha g_{AB}^{\mathrm{ST}}(t) = D(p_B^{\mathrm{ST}}(t)), \tag{142}$$

$$g_{AB}^{\mathrm{ST}}(t) = \frac{\alpha p_B^{\mathrm{ST}}(t)}{m_A + q}, \tag{143}$$

$$b_A^{\mathrm{ST}}(t) = D(c + \lambda + \tau). \tag{144}$$

Differentiating (141) - (143) yields $\frac{\partial p_B^{\mathrm{ST}}}{\partial \lambda} = 0$. □

The transversality conditions for $E(t)$, $\lambda(t)$ and $S(t)$ are

$$(a)\ \lim_{t\to\infty} e^{-\rho t}\tilde{\theta}_A(t)\left[E(t) - E^{opt}(t)\right] \geq 0, \qquad (b)\ \lim_{t\to\infty} e^{-\rho t}\mu(t)\left[\lambda(t) - \lambda^{opt}(t)\right] \geq 0, \tag{145}$$

$$\lim_{t\to\infty} e^{-\rho t}\eta(t)\left[S(t) - S^{opt}(t)\right] \geq 0. \tag{146}$$

Solving (112) and taking account of (145)(a) yields

$$\tilde{\theta}_A(t) = -\frac{h}{\rho + \gamma}, \tag{147}$$

while (113) implies

$$\eta(t) = \eta_0 e^{\rho t}. \tag{148}$$

Integrating (111) gives

$$\begin{aligned} \mu(t) = \alpha \int_0^t \left[g_{BA}^s(j)\frac{\partial p_A(j)}{\partial \lambda(j)} - g_{AB}^s(j)\frac{\partial p_B(j)}{\partial \lambda(j)}\right] dj + \int_0^t \frac{h}{\rho+\gamma}\frac{\partial b_B^d(j)}{\partial \lambda(j)}dj \\ + \left\{\int_0^t b_A^d(j)dj - s_A\int_0^t [b_A^d(j) + b_B^d(j)]dj\right\} + [\eta_0 - s_A\lambda_0]\int_0^t e^{\rho j}\frac{\partial b_B^d(j)}{\partial \lambda(j)}dj. \end{aligned} \tag{149}$$

For $t \geq T$, the value function of country A reads $\tilde{V} = \int_0^\infty e^{-\rho t}\Big[U(g_A^d(t)) + \bar{\ell}_A + \alpha\left[p_B(t)g_{AB}^s(t) - p_A(t)g_{BA}^s(t)\right] - M_A(g_{AA}^s(t) + g_{AB}^s(t)) - Q(g_{AB}^s(t)) - hE(T)e^{-\gamma t}\Big]dt$, where $\bar{U} := U(g_A^d(t)) + \bar{\ell}_A + \alpha\left[p_B(t)g_{AB}^s(t) - p_A(t)g_{BA}^s(t)\right] - M_A(g_{AA}^s(t) + g_{AB}^s(t)) - Q(g_{AB}^s(t))$ is a constant. Solving yields

$$\tilde{V} = \frac{\bar{U}}{\rho} - \frac{h}{\rho+\gamma}E(T). \tag{150}$$

The point in time T is determined by the transversality condition $H(T) = \rho\tilde{V}$, where $H(T)$ denotes the value of the Hamiltonian evaluated at time T. The condition gives

$$\mu(T)\rho\lambda(T) = 0. \tag{151}$$

By substituting (149) we get

$$
\begin{aligned}
\eta_0 = s_A\lambda_0 - \frac{1}{\int_0^T e^{\rho j}\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj}\Bigg\{\alpha\int_0^T\left[g_{BA}^s(j)\frac{\partial p_A(j)}{\partial\lambda(j)} - g_{AB}^s(j)\frac{\partial p_B(j)}{\partial\lambda(j)}\right]dj \\
+ \int_0^T \frac{h}{\rho+\gamma}\frac{\partial b_B^d(j)}{\partial\lambda(j)}dj + \left[\int_0^T b_A^d(j)dj - s_A S_0\right]\Bigg\}.
\end{aligned}
\tag{152}
$$

Substituting into (110) yields (42) for an interior solution.

A.6 Calibration

See table 1.

Region	Countries
Europe	Albania, Austria, Belgium, Bosnia and Herzegovina, Bulgaria, Croatia, Cyprus, Czechia, Denmark, Estonia, Faroe Islands, Finland, France, Germany, Gibraltar, Greece, Hungary, Iceland, Ireland, Italy, Kosovo, Latvia, Lithuania, Luxembourg, Malta, Moldova, Montenegro, Netherlands, North Macedonia, Norway, Poland, Portugal, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, Ukraine, United Kingdom
Eurasia	Armenia, Azerbaijan, Belarus, Georgia, Kazakhstan, Kyrgyzstan, Russia, Tajikistan, Turkmenistan, Uzbekistan
MENA	Algeria, Bahrain, Egypt, Iran, Iraq, Israel, Jordan, Kuwait, Lebanon, Libya, Morocco, Oman, Palestinian Territories, Qatar, Saudi Arabia, Syria, Tunisia, United Arab Emirates, Yemen
SSA	Angola, Benin, Botswana, Burkina Faso, Burundi, Cabo Verde, Cameroon, Central African Republic, Chad, Comoros, Congo-Brazzaville, Congo-Kinshasa, Cote d'Ivoire, Djibouti, Equatorial Guinea, Eritrea, Eswatini, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Nigeria, Rwanda, Saint Helena, Sao Tome and Principe, Senegal, Seychelles, Sierra Leone, Somalia, South Africa, South Sudan, Sudan, Tanzania, Togo, Uganda, Western Sahara, Zambia, Zimbabwe
SAARC	Afghanistan, Bangladesh, Bhutan, India, Maldives, Nepal, Pakistan, Sri Lanka
NE Asia	China (mainland), Japan, Mongolia, North Korea, South Korea
SE Asia	America Samoa, Australia, Brunei, Burma, Cambodia, Cook Islands, Fiji, French Polynesia, Guam, Hong Kong, Indonesia, Kiribati, Laos, Malaysia, Macau, Micronesia, Nauru, New Caledonia, New Zealand, Niue, Papua New Guinea, Philippines, Samoa, Singapore, Solomon Islands, Taiwan, Thailand, Timor-Leste, Tonga, Tuvalu, Vanuatu, Vietnam, Wake Island
N. America	Bahamas, Bermuda, Canada, Cayman Islands, Cuba, Dominican Republic, Greenland, Haiti, Mexico, Saint Pierre and Miquelon, Turks and Caicos Islands, United States
S. America	Antarctica, Antigua and Barbuda, Argentina, Aruba, Barbados, Belize, British Virgin Islands, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominica, Ecuador, El Salvador, Falkland Islands, Grenada, Guyana, Guatemala, Honduras, Jamaica, Montserrat, Netherlands Antilles, Nicaragua, Panama, Paraguay, Peru, Puerto Rico, Saint Kitts and Nevis, Saint Lucia, Saint Vincent/Grenadines, Suriname, Trinidad and Tobago, Uruguay, U.S. Virgin Islands Venezuela

Table 1: The nine world regions