

B Online Appendix

Definition 2

- t_c : If all green energy production of country B is exported to country A at $t = 0$, t_c marks the point in time domestic green energy use becomes profitable in country B .
- t_d : The point in time green energy exports from country B to country A ends is denoted by t_d .
- t_e : The point in time green energy exports from country A to country B ends is denoted by t_e .
- t_f : If all green energy production of country A is exported to country B at $t = 0$, t_f marks the point in time domestic green energy use becomes profitable in country A .
- T_A^i : If fossil fuel use ends in country A more than once, T_A^i denotes the point in time that fuel use ends the $i = 1, 2$ time.
- T_B^i : If fossil fuel use ends in country A more than once, T_B^i denotes the point in time that fuel use ends the $i = 1, 2$ time.

Define $\tilde{\xi}$ as the corresponding counter-point in time to $\xi \in \{t_c, t_d, t_e, t_f, T_A^i, T_B^i\}$, e.g. $\tilde{t}_c$ denotes the point in time, country B starts to export all its green energy to country A . In the following, we check in which sequence $t_c, \tilde{t}_c, t_d, \tilde{t}_d, t_e, \tilde{t}_e, t_f, \tilde{t}_f, T_A^i, \tilde{T}_A, T_B^i$, and $\tilde{T}_B$ can appear.

Lemma 4 Let $\xi \in \{t_c, t_d, t_e, t_f, T_A^i, T_B^i\}$ and $\tilde{\xi}$ the corresponding counter-point in time. Then, ξ and $\tilde{\xi}$ cannot directly follow on each other in a sequence.

Proof Suppose that either ξ directly follows $\tilde{\xi}$ or that $\tilde{\xi}$ directly follows ξ implying $\xi > \tilde{\xi}$ or $\xi < \tilde{\xi}$, respectively. Then, the system of equations following from (??), (??) and (??) describing the market equilibrium at $t = \xi$ is identical to the system of equations describing the market equilibrium at $t = \tilde{\xi}$. However, this system has a unique solution in t implying $\xi = \tilde{\xi}$. The contradiction proofs the lemma.* $\square$

B.1 Timings with initial trade - Proof of Lemmata 5 and 6

Lemma 5 Suppose that green energy is traded initially, that fossil fuel is consumed in both countries initially and that country A does not act strategically. In the unilaterally regulated

*Suppose that $g_{BB} > 0$ and the previous element in the sequence was T_A . If $\tilde{T}_A$ follows, we get $p_A(t) = M'_A(g_{AA}^s(t)) = U'(g_{AA}^s(t) + \alpha g_{BA}^s(t)) = c + \tau + \lambda^{ST}(t)$, $\alpha p_A(t) = c + \lambda(t) + Q'(g_{BA}^s(t))$. Solving for t yields $t = \frac{1}{\rho} \ln \left(\frac{aqm_A - [\alpha^2 m_A + q + qzm_A][c + \tau] + \alpha m_A c}{[\alpha^2 m_A + q + qzm_A]\lambda_0^{ST} - \alpha m_A \lambda_0} \right)$ which is unique.

economy the timing is given by

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| i) $0 < t_c < T_A < T_B$, | x) $0 < T_A < T_B$, |
| ii) $0 < t_c < T_A < t_d < T_B$, | xi) $0 < T_A < t_d < T_B$, |
| iii) $0 < t_c < T_B < T_A$, | xii) $0 < T_A < t_c < t_d < T_B$, |
| iv) $0 < t_c < t_d < T_A < T_B$, | xiii) $0 < T_A < t_c < T_B$, |
| v) $0 < t_c < t_d < T_B < \tilde{t}_d < T_A$, | xiv) $0 < T_A^1 < t_c < \tilde{T}_A < T_B < T_A^2$, |
| vi) $0 < t_c < t_d < T_B < T_A$, | xv) $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_A^2 < T_B$, |
| vii) $0 < t_d < T_A < T_B$, | xvi) $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < T_A^2$, |
| viii) $0 < t_d < T_B < T_A$, | xvii) $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < T_A^2$, |
| ix) $0 < t_d < T_B < \tilde{t}_d < T_A$, | xviii) $0 < T_B < T_A$. |

Lemma 6 Suppose that green energy is traded initially, fossil fuel is consumed in both countries initially and country A acts strategically. In the unilaterally regulated economy with the fossil fuel tax τ^{ST} in country A and no fuel tax in country B the timing is given by

(a) If $SE > 0$, the timings are given by Lemma 5 and

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| i) $0 < t_c < t_d < T_B^1 < \tilde{t}_d < \tilde{T}_B < T_A < T_B^2$, | iv) $0 < \tilde{t}_c < T_A < t_c < T_B$, |
| ii) $0 < t_d < T_B^1 < \tilde{t}_d < \tilde{T}_B < T_A < T_B^2$, | v) $0 < \tilde{t}_c < T_A < t_c < t_d < T_B$. |
| iii) $0 < \tilde{t}_c < T_A^1 < t_c < \tilde{T}_A < T_B < T_A^2$, | |

(b) If $SE < 0$ and $g_{BA}(0) > 0$, the timings are given by Lemma 5 and

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| i) $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A$, |
| ii) $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A$, |
| iii) $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < T_A$, |
| iv) $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A$, |
| v) $0 < t_d < \tilde{t}_e < T_B < t_e < T_A$, |
| vi) $0 < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A$, |
| vii) $0 < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A$, |
| viii) $0 < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A$, |
| ix) $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < T_B < t_e < T_A^2$, |
| x) $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A^2$, |
| xi) $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A^2$, |
| xii) $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A^2$. |

(c) If $SE < 0$ and $g_{AB}(0) > 0$, the timings are given by

$$\begin{aligned}
i) & 0 < t_e < T_A < T_B, & x) & 0 < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A, \\
ii) & 0 < t_e < T_B < T_A, & xi) & 0 < T_B < t_e < T_A, \\
iii) & 0 < t_e < T_B < \tilde{t}_d < T_A, & xii) & 0 < T_B < t_e < \tilde{t}_d < T_A, \\
iv) & 0 < t_f < t_e < T_A < T_B, & xiii) & 0 < T_B < t_f < t_e < T_A, \\
v) & 0 < t_f < t_e < T_B < T_A, & xiv) & 0 < T_B < t_f < t_e < \tilde{t}_d < T_A, \\
vi) & 0 < t_f < t_e < T_B < \tilde{t}_d < T_A, & xv) & 0 < T_B^1 < t_f < \tilde{T}_B < t_e < T_A < T_B^2, \\
vii) & 0 < t_f < T_B < t_e < T_A, & xvi) & 0 < T_B^1 < t_f < \tilde{T}_B < t_e < T_B^2 < T_A, \\
viii) & 0 < t_f < T_B < t_e < \tilde{t}_d < T_A, & xvii) & 0 < T_B^1 < t_f < \tilde{T}_B < t_e < T_B^2 < \tilde{t}_d < T_A, \\
ix) & 0 < \tilde{t}_f < T_B < t_f < t_e < T_A, & &
\end{aligned}$$

B.1.1 $SE \geq 0$

At first, suppose that the strategic effect is non-negative. The assumption $b_A(0), b_B(0), g_{BA}(0) > 0$ rule out that $\tilde{T}_A, \tilde{T}_B, \tilde{t}_d, t_e, \tilde{t}_e, t_f$, and $\tilde{t}_f$ are the first element of a sequence following the initial point in time $t = 0$. In case of $\tilde{t}_c$, $g_{BB}(t) > 0$ has to hold before $\tilde{t}_c$, so that the price in country A has to grow faster than in country B to render the export of the complete green energy production profitable. Without the strategic effect, $\dot{p}_A = \rho\lambda(t) = \dot{p}_B$. Consequently, $\tilde{t}_c$ cannot be the first element in case of Lemma 5.

Case $[0, t_c)$

Suppose that t_c is the first element. Then, for $t \in [0, t_c)$ we get from (20), (51) and (52)

$$p_A(t) = M'_A(g_{AA}^s(t)) = U'(b_A^d(t) + g_{AA}^s(t) + \alpha g_{BA}^s(t)) = c + \tau^U + \lambda(t)^{ST}, \quad (153)$$

$$p_B(t) = U'(b_B^d(t)) = c + \lambda(t), \quad (154)$$

$$\alpha p_A(t) = M'_B(g_{BA}^s(t)) + Q'(g_{BA}^s(t)). \quad (155)$$

Differentiating with respect to time yields

$$\dot{p}_A = \rho\lambda^{ST}(t) > 0, \quad \dot{p}_B = \rho\lambda(t) > 0, \quad (156)$$

$$\dot{g}_{AA}^s = \frac{\rho\lambda^{ST}(t)}{M_A''} > 0, \quad \dot{b}_A^d(t) = \frac{\rho\lambda^{ST}(t)}{U''} - \frac{\rho\lambda^{ST}(t)}{M_A''} - \frac{\alpha^2\rho\lambda^{ST}(t)}{M_B'' + Q''} < 0, \quad (157)$$

$$\dot{g}_{BA}^s = \frac{\alpha\rho\lambda^{ST}(t)}{M_B'' + Q''} > 0, \quad \dot{b}_B^d(t) = \frac{\rho\lambda(t)}{U''} < 0. \quad (158)$$

Let denote χ the next element of the sequence. Then, for $t \in [t_c, \chi)$ we get (153) and

$$p_B(t) = M'_B(g_{BB}^s(t) + g_{BA}^s(t)) = U'(b_B^d(t) + g_{BB}^s(t)) = c + \lambda(t), \quad (159)$$

$$\alpha p_A(t) = p_B(t) + Q'(g_{BA}^s(t)). \quad (160)$$

Differentiating yields (156),

$$\dot{g}_{AA}^s = \frac{\rho\lambda^{\text{ST}}(t)}{M_A''} > 0, \quad \dot{b}_A^d(t) = \rho\lambda^{\text{ST}}(t) \left[\frac{1}{U''} - \frac{1}{M_A''} \right] - \alpha\dot{g}_{BA}^s \leq 0 \quad (161)$$

$$\dot{g}_{BB}^s = \frac{\rho\lambda(t)}{M_B''} - \dot{g}_{BA}^s \leq 0, \quad (162)$$

$$\dot{g}_{BA}^s = \frac{\alpha\rho\lambda^{\text{ST}}(t) - \rho\lambda(t)}{Q''} \leq 0, \quad \dot{b}_B^d(t) = \rho\lambda(t) \left[\frac{1}{U''} - \frac{1}{M_B''} \right] + \dot{g}_{BA}^s \leq 0. \quad (163)$$

Without green energy trade, we get $\dot{g}_{BB}^s = \frac{\rho\lambda}{M_B''} > 0$ for $t \in [0, t_c)$. t_c can only be the first element of the sequence, if $\dot{g}_{BB}^s > \dot{g}_{BA}^s$ holds implying $\frac{\alpha\rho\lambda^{\text{ST}} - \rho\lambda}{Q''} < \frac{\rho\lambda}{M_B''}$, so that $\dot{g}_{BB}^s > 0$ and $\dot{b}_B^d < 0$ for $t \in [t_c, \chi)$.

Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because of Lemma 4, $\chi \neq \tilde{t}_c$. Because $g_{BA}^s(t) > 0$, $\chi \notin \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \notin \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \notin \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = \{t_d, T_A, T_B\}$.

Case $[0, t_c), [t_c, T_A)$ Suppose that $0 < t_c < T_A < \dots$. Then, for $t \in [T_A, \chi)$ we get (159), (160) and

$$p_A(t) = M'_A(g_{AA}^s(t)) = U'(g_{AA}^s(t) + \alpha g_{BA}^s(t)). \quad (164)$$

Differentiating with respect to time yields

$$\dot{p}_A = \frac{M_A'' U'' \alpha \rho \lambda(t)}{\alpha^2 M_A'' U'' - Q''[M_A'' - U'']} > 0, \quad \dot{p}_B = \rho\lambda(t) > 0, \quad (165)$$

$$\dot{g}_{AA}^s = \frac{U'' \alpha \rho \lambda(t)}{\alpha^2 M_A'' U'' - Q''[M_A'' - U'']} > 0, \quad (166)$$

$$\dot{g}_{BB}^s = \rho\lambda(t) \left[\frac{1}{M_B''} - \frac{M_A'' - U''}{\alpha^2 M_A'' U'' - Q''[M_A'' - U'']} \right] > 0, \quad (167)$$

$$\dot{g}_{BA}^s = \frac{[M_A'' - U''] \rho \lambda(t)}{\alpha^2 M_A'' U'' - Q''[M_A'' - U'']} < 0, \quad (168)$$

$$\dot{b}_B^d = \rho\lambda(t) \left[\frac{1}{U''} - \frac{1}{M_B''} + \frac{M_A'' - U''}{\alpha^2 M_A'' U'' - Q''[M_A'' - U'']} \right] < 0. \quad (169)$$

Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $\dot{g}_{BB}^s(t) > 0$ and $\dot{g}_{BA}^s(t) < 0$, $\chi \neq \tilde{t}_c$. Because $g_{BA}^s(t) > 0$, $\chi \notin \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \notin \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) = 0$, $\chi \neq T_A$. Because of Lemma 4, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) > 0$, $\chi \neq \tilde{T}_B$. Thus, $\chi = \{t_d, T_B\}$.

Case $[0, t_c), [t_c, T_A), [T_A, T_B)$ Suppose that $0 < t_c < T_A < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (160), (164) and

$$p_B(t) = M'_B(g_{BB}^s(t) + g_{BA}^s(t)) = U'(g_{BB}^s(t)). \quad (170)$$

The solution of this equation system does not depend on time, so that T_B is the last element of the sequence given by $0 < t_c < T_A < T_B$.

Case $[0, t_c), [t_c, T_A), [T_A, t_d)$ Suppose that $0 < t_c < T_A < t_d < \dots$. Then, for $t \in [t_d, \chi)$ we get

$$p_A(t) = M'_A(g_{AA}^s(t)) = U'(g_{AA}^s(t)) = \bar{p}_A, \quad (171)$$

$$p_B(t) = M'_B(g_{BB}^s(t)) = U'(b_B^d(t) + g_{BB}^s(t)) = c + \lambda(t). \quad (172)$$

Differentiating yields

$$\dot{p}_A = \dot{g}_{AA}^s = 0, \quad \dot{p}_B = \rho\lambda(t) > 0 \quad (173)$$

$$\dot{g}_{BB}^s = \frac{\rho\lambda(t)}{M_B''} > 0, \quad \dot{b}_B^d = \frac{\rho\lambda(t)}{U''} - \frac{\rho\lambda(t)}{M_B''} < 0. \quad (174)$$

Because $g_{BA}^s(t) = g_{AB}^s(t) = 0$, $\chi \notin \{t_c, \tilde{t}_c, t_d, t_e, t_f, \tilde{t}_f\}$. Because $\alpha\dot{p}_A(t) - \dot{p}_B(t) < 0$, $\chi \neq \tilde{t}_d$. Because $\bar{p}_A > \bar{p}_B > p_B(t)$, $\chi \neq \tilde{t}_e$. Because $b_A^d(t) = 0$, $\chi \neq T_A$. Because $\dot{p}_A < \rho\lambda^{\text{ST}}$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) > 0$, $\chi \neq \tilde{T}_B$. Thus, $\chi = T_B$. For $t \geq T_B$ we get (171) and

$$p_B(t) = M'_B(g_{BB}^s(t)) = U'(g_{BB}^s(t)) = \bar{p}_B. \quad (175)$$

Because the solution of this equation system does not depend on time, we get the sequence $0 < t_c < T_A < t_d < T_B$.

Case $[0, t_c), [t_c, T_B)$ Suppose $0 < t_c < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (153), (160) and (170). Differentiating with respect to time yields

$$\dot{p}_A = \rho\lambda^{\text{ST}}(t) > 0, \quad \dot{p}_B = \frac{U''M_B''\alpha\rho\lambda^{\text{ST}}(t)}{Q''[U'' - M_B''] + U''M_B''} \in (0, \alpha\rho\lambda^{\text{ST}}(t)) < \rho\lambda^{\text{ST}}(t), \quad (176)$$

$$\dot{g}_{AA}^s = \frac{\rho\lambda^{\text{ST}}(t)}{M_A''} > 0, \quad \dot{b}_A^d = \rho\lambda^{\text{ST}}(t) \left[\frac{1}{U''} - \frac{1}{M_A''} - \frac{\alpha^2[U'' - M_B'']}{Q''[U'' - M_B''] + U''M_B''} \right] < 0, \quad (177)$$

$$\dot{g}_{BB}^s = \frac{M_B''\alpha\rho\lambda^{\text{ST}}(t)}{Q''[U'' - M_B''] + U''M_B''} < 0, \quad (178)$$

$$\dot{g}_{BA}^s = \frac{[U'' - M_B'']\alpha\rho\lambda^{\text{ST}}(t)}{Q''[U'' - M_B''] + U''M_B''} > 0. \quad (179)$$

Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $b_B^d(t) = 0$, $\chi \neq \tilde{t}_c$. Because $\dot{g}_{BA}^s > 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \notin \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \notin \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Because of Lemma 4, $\chi \neq \tilde{T}_B$. Thus, $\chi = T_A$. For $t \geq T_A$ we get (160), (164) and (170), with a solution that does not depend on time. Therefore, the sequence reads $0 < t_c < T_B < T_A$.

Case $[0, t_c), [t_c, t_d)$ Suppose that $0 < t_c < t_d < \dots$. Then, for $t \in [t_d, \chi)$ we get (172) and

$$p_A(t) = M'_A(g_{AA}^s(t)) = U'(b_A^d(t) + g_{AA}^s(t)) = c + \tau + \lambda(t). \quad (180)$$

Differentiating with respect to time yields (156), (174) and

$$\dot{g}_{AA}^s = \frac{\rho\lambda^{\text{ST}}(t)}{M_A''} > 0, \quad \dot{b}_A^d = \frac{\rho\lambda^{\text{ST}}(t)}{U''} - \frac{\rho\lambda^{\text{ST}}(t)}{M_A''} < 0. \quad (181)$$

Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because of Lemma 4, $\chi \neq \tilde{t}_d$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $\alpha\dot{p}_B - \dot{p}_A < 0$, $\chi \neq \tilde{t}_e$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = \{T_A, T_B\}$.

Case $[0, t_c), [t_c, t_d), [t_d, T_A)$ Suppose that $0 < t_c < t_d < T_A < \dots$. Then, for $t \in [T_A, \chi)$ we get (171) and (172). Differentiating with respect to time yields (173) and (174). Applying the related argumentation yields the sequence $0 < t_c < t_d < T_A < T_B$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B)$ Suppose that $0 < t_c < t_d < T_B < \dots$. Then, for $t \in [T_b, \chi)$ we get (180) and (175). Differentiating yields (181),

$$\dot{p}_A = \rho\lambda^{\text{ST}}(t) > 0, \quad \dot{p}_B = 0, \quad (182)$$

$$\dot{g}_{BB}^s = \dot{b}_B^d = 0. \quad (183)$$

Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $\alpha\dot{p}_B - \dot{p}_A < 0$, $\chi \neq \tilde{t}_e$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Because $\dot{p}_B < \rho\lambda$, $\chi \neq \tilde{T}_B$. Thus, $\chi = \{\tilde{t}_d, T_A\}$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B), [T_B, T_A)$ Suppose that $0 < t_c < t_d < T_B < T_A < \dots$. Then, for $t \in [T_A, \chi)$ we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_c < t_d < T_B < T_A$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B), [T_B, \tilde{t}_d)$ Suppose that $0 < t_c < t_d < T_B < \tilde{t}_d < \dots$. Then, for $t \in [\tilde{t}_d, \chi)$ we get (153), (170) and (160). Differentiating yields (176) - (179). Because

$g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $b_B^d(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because $\dot{g}_{BA}^s > 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Thus, $\chi = \{T_A, \tilde{T}_B\}$. In case of $SE = 0$, also $\chi \neq \tilde{T}_B$, because $\dot{p}_B < \rho\lambda$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, T_A)$ Suppose that $0 < t_c < t_d < T_B < \tilde{t}_d < T_A < \dots$. Then, for $t \in [T_A, \chi)$ we get (164), (170) and (160). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_c < t_d < T_B < \tilde{t}_d < T_A$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B)$ Suppose that $0 < t_c < t_d < T_B < \tilde{t}_d < \tilde{T}_B < \dots$. Then, for $t \in [\tilde{T}_B, \chi)$ we get (153), (159) and (160), so that (156) and (161) - (163) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because the equation system describing the market equilibrium is identical to the one for $t \in [t_c, t_d)$, the proof of Lemma 4 can be applied, so that $\chi \neq \{\tilde{t}_c, t_d\}$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because of Lemma 4, $\chi \neq T_B$. Thus, $\chi = T_A$.

For $t \in [T_A, \chi)$, we get (164), (159) and (160). Differentiating with respect to time yields (165) - (169). Following the related argument yields $\chi = \{t_d, T_B\}$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B^1), [T_B^1, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, T_A), [T_A, T_B^2)$ Suppose that $0 < t_c < t_d < T_B^1 < \tilde{t}_d < \tilde{T}_B < T_A < T_B^2 < \dots$. Then, for $t \in [T_B^2, \chi)$ we get (164), (170) and (160). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_c < t_d < T_B^1 < \tilde{t}_d < \tilde{T}_B < T_A < T_B^2$.

Case $[0, t_c), [t_c, t_d^1), [t_d^1, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, T_A), [T_A, t_d^2)$ Suppose that $0 < t_c < t_d^1 < T_B < \tilde{t}_d < \tilde{T}_B < T_A < t_d^2 < \dots$. Then, for $t \in [t_d^2, \chi)$ we get (171) and (172), so that (173) and (174) hold. Following the related arguments yields $\chi = T_B$ suggesting the sequence $0 < t_c < t_d^1 < T_B^1 < \tilde{t}_d < \tilde{T}_B < T_A < t_d^2 < T_B^2$. Because (175) determines both T_B^1 and T_B^2 , the proof of Lemma 4 applies implying that this case is not possible.

Case $[0, t_d)$

Suppose that t_d is the first element. Then, for $t \in [0, t_d)$ we get (153), (159) and (160) from (20), (51) and (52). Differentiating with respect to time yields (156), (161) - (163), with $\dot{g}_{BA}^s < 0$, $\dot{g}_{BB}^s > 0$ and $\dot{b}_B < 0$, because t_d is the first element.

For $t \in [t_d, \chi)$ we get (180) and (172). Differentiating yields (156), (174) and (181). Because $g_{BA}^s(t) = g_{AB}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d, t_e, t_f, \tilde{t}_f\}$. Because of Lemma 4, $\chi \neq \tilde{t}_d$. Because $\alpha\dot{p}_B - \dot{p}_A < 0$, $\chi \neq \tilde{t}_e$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = \{T_A, T_B\}$.

Case $[0, t_d), [t_d, T_A)$ Suppose that $0 < t_d < T_A < \dots$. Then, for $t \in [T_A, \chi)$ we get (171) and (172), so that (173) and (174) hold. Following the related argument yields $\chi = T_B$ and, therefore, the sequence $0 < t_d < T_A < T_B$.

Case $[0, t_d), [t_d, T_B)$ Suppose that $0 < t_d < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (180) and (175). Differentiating yields (181) - (183). Following the related argument gives $\chi = \{\tilde{t}_d, T_A\}$.

Case $[0, t_d), [t_d, T_B), [T_B, T_A)$ Suppose that $0 < t_d < T_B < T_A < \dots$. Then, for $t \in [T_A, \chi)$ we get (171) and (175). Because the solution of the equation does not depend on time, we get the sequence $0 < t_d < T_B < T_A$.

Case $[0, t_d), [t_d, T_B), [T_B, \tilde{t}_d)$ Suppose that $0 < t_d < T_B < \tilde{t}_d < \dots$. Then, for $t \in [\tilde{t}_d, \chi)$ we get (153), (170) and (160). Differentiating yields (176) - (179). Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $b_B^d(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because $\dot{g}_{BA}^s > 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Thus, $\chi = \{T_A, \tilde{T}_B\}$. In case of $SE = 0$, also $\chi \neq \tilde{T}_B$, because $\dot{p}_B < \rho\lambda$.

Case $[0, t_d), [t_d, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, T_A)$ Suppose that $0 < t_d < T_B < \tilde{t}_d < T_A < \dots$. Then, for $t \in [T_A, \chi)$ we get (164), (170) and (160). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_d < T_B < \tilde{t}_d < T_A$.

Case $[0, t_d), [t_d, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B)$ Suppose that $0 < t_d < T_B < \tilde{t}_d < \tilde{T}_B < \dots$. Then, for $t \in [\tilde{T}_B, \chi)$ we get (153), (159) and (160), so that (156) and (161) - (163) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $\dot{g}_{BA}^s < 0$, $\chi \neq \tilde{t}_c$. Because the equation system describing the market equilibrium is identical to the one for $t \in [0, t_d)$, the proof of Lemma 4 can be applied, so that $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because of Lemma 4, $\chi \neq T_B$. Thus, $\chi = T_A$.

For $t \in [T_A, \chi)$, we get (164), (159) and (160). Differentiating with respect to time yields (165) - (169). Following the related argument yields $\chi = \{t_d, T_B\}$. The remaining argument was already used for the cases $\{[0, t_c), [t_c, t_d), [t_d, T_B^1), [T_B^1, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, T_A), [T_A, T_B^2)\}$ and $\{[0, t_c), [t_c, t_d^1), [t_d^1, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, T_A), [T_A, t_d^2)\}$, which implies the sequence $0 < t_d < T_B^1 < \tilde{t}_d < \tilde{T}_B < T_A < T_B^2$.

Case $[0, T_A)$, with $g_{BB}^s(0) > 0$

Suppose that T_A is the first element and that $g_{BB}(t) > 0$ holds before T_A . Then, for $t \in [0, T_A)$ we get (153), (159) and (160). Consequently, the dynamics are described by (156) and (161) - (163), with $\dot{b}_A^d < 0$.

For $t \in [T_A, \chi)$ we get (164), (159) and (160), so that (165) - (169) hold. Following the related argument yields $\chi = \{t_d, T_B\}$.

Case $[0, T_A), [T_A, T_B)$ Suppose that $0 < T_A < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (164), (170) and (160). Because the solution of the equation system does not depend on time, we get the sequence $0 < T_A < T_B$.

Case $[0, T_A), [T_A, t_d)$ Suppose that $0 < T_A < t_d < \dots$. Then, for $t \in [t_d, \chi)$ we get (171) and (172), so that (173) and (174) hold. Following the related argument yields the sequence $0 < T_A < t_d < T_B$.

Case $[0, T_A)$, with $g_{BB}^s(0) = 0$

Suppose that T_A is the first element and that $g_{BB}(t) = 0$ and $g_{BA}^s(t) > 0$ hold before T_A . For $t \in [0, T_A)$, we get (153) - (155), so that the evolution is given by (156) - (158).

For $t \in [T_A, \chi)$, we get (164), (154) and (155). Differentiating with respect to time yields (173) and

$$\dot{b}_B^d = \frac{\rho\lambda(t)}{U''} < 0, \quad \dot{g}_{BA}^s = 0. \quad (184)$$

Because $g_{BB}^s(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) = 0$, $\chi \neq T_A$. Because $\dot{p}_A < \rho\lambda^{\text{ST}}$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) > 0$, $\chi \neq \tilde{T}_B$. Thus, $\chi = \{t_c, t_d\}$.

In case of $\chi = t_d$, the end of trade implies $g_{BB}^s(t) > 0$, so that $t_d = t_c$ holds. Then, for $t \in [t_d, \chi)$ we get (171) and (172), with (173) and (174) describing the evolution over time. Following the related argument yields the sequence $0 < T_A < t_d < T_B$.

Case $[0, T_A), [T_A, t_c)$ Suppose that $0 < T_A < t_c < \dots$. Then, for $t \in [t_c, \chi)$ we get (164), (159) and (160), so that (165) - (169) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because of Lemma 4, $\chi \neq \tilde{t}_c$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) = 0$, $\chi \neq T_A$. Because $b_B^d(t) > 0$, $\chi \neq \tilde{T}_B$. Thus, $\chi = \{t_d, \tilde{T}_A, T_B\}$.

Case $[0, T_A), [T_A, t_c), [t_c, t_d)$ Suppose that $0 < T_A < t_c < t_d < \dots$. Then, for $t \in [t_d, \chi)$ we get (171) and (172), so that (173) and (174) hold. Following the related argument yields the sequence $0 < T_A < t_c < t_d < T_B$.

Case $[0, T_A), [T_A, t_c), [t_c, T_B)$ Suppose that $0 < T_A < t_c < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (164), (170) and (160). Because the solution of this equation system does not depend on time, we get the sequence $0 < T_A < t_c < T_B$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < \dots$. Then, for $t \in [\tilde{T}_A, \chi)$ we get (153), (159) and (160), so that (156) and (161) - (163) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because of Lemma 4, $\chi \neq T_A$. Thus, $\chi = \{\tilde{t}_c, t_d, T_B\}$.

Suppose that $0 < T_A < t_c < \tilde{T}_A < \tilde{t}_c < \dots$. Then, for $t \in [\tilde{t}_c, \chi)$ we get (153) - (155), so that (156) - (158) hold. Because of Lemma 4, $\chi \neq t_c$. Because $g_{BB}^s(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because $\dot{g}_{BA}^s > 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because the equations system is identical to the one of $t \in [0, T_A)$, the proof of Lemma 4 can be applied, so that $\chi \neq T_A$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Consequently, $\tilde{t}_c$ cannot follow $\tilde{T}_A$ implying $\chi = \{t_d, T_B\}$ for $0 < T_A < t_c < \tilde{T}_A < \dots$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, T_B)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (153), (170) and (160), so that (176) - (179) hold. Following the related argument yields the sequence $0 < T_A^1 < t_c < \tilde{T}_A < T_B < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < \dots$. Then, for $t \in [t_d, \chi)$ we get (180) and (172), so that (156), (174) and (181) hold. Following the related argument yields $\chi = \{T_A, T_B\}$.

Case $[0, T_A^1), [T_A^1, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_A^2)$ Suppose that $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_A^2 < \dots$. Then, for $t \in [T_A^2, \chi)$ we get (171) and (172), so that (173) and (174) hold implying the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_A^2 < T_B$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_B)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (180) and (175), so that (181) - (183) hold. Following the related argument yields $\chi = \{\tilde{t}_d, T_A\}$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_B), [T_B, T_A)$ Suppose that $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < T_A^2 < \dots$. Then, for $t \in [T_A^2, \chi)$ we get (171) and (175). Because the solution of this equation system does not depend on time, we get the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_B), [T_B, \tilde{t}_d)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < \dots$. Then, for $t \in [\tilde{t}_d, \chi)$ we get (153), (170) and (160), so that (176) - (179) hold. Following the argument from the case $0 < t_d < T_B < \tilde{t}_d < \dots$ yields $\chi = \{T_A, \tilde{T}_B\}$.

Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < \tilde{T}_B < \dots$. Then, for $t \in [\tilde{T}_B, \chi)$ we get (153), (159) and (160), so that (156) and (161) - (179) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because of Lemma 4, $\chi \neq T_B$. If $\chi = t_d^2$, the equation system at t_d^2 is identical to the one at $t = t_d^1$. Applying the proof of Lemma 4 implies $\chi \neq t_d^2$. Analogously, the proof rules out $\chi = \tilde{T}_A$. Thus, $\chi = \tilde{t}_c$ implying $0 < T_A < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < \tilde{T}_B < \tilde{t}_c < \dots$.

For $t \in [\tilde{t}_c, \chi)$ we get (153) - (155), so that (156) - (158) hold. Because of Lemma 4, $\chi \neq t_c$. Because $g_{BB}^s(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because $\dot{g}_{BA}^s > 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. If $\chi = T_A^2$, the equation system describing the market equilibrium is identical to the one at $t = T_A^1$. Due to the proof of Lemma 4, $\chi \neq T_A$, which rules out that $\tilde{T}_B$ follows $\tilde{t}_d$ for $0 < T_A < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < \dots$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, T_A)$ Suppose that $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < T_A^2 < \dots$. Then, for $t \in [T_A^2, \chi)$ we get (164), (170) and (160). Because the solution of this equation system does not depend on time, we get the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < T_A^2$.

Case $[0, T_B)$

Suppose that T_B is the first element. Then, $g_{BB}^s(t) > 0$ for $t \in [0, T_B)$ to ensure $y_B(T_B) > 0$. For $t \in [0, T_B)$, we get (153), (159) and (160), with (156), (161) - (163) describing the evolution over time.

For $t \in [T_B, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Following the related argument yields the sequence $0 < T_B < T_A$.

Case $[0, \tilde{t}_c)$

If $SE > 0$, the first element of the sequence can be $\tilde{t}_c$. For $t \in [0, \tilde{t}_c)$ we get $g_{BB}^s(t) > 0$ implying (153), (159) and (160). Consequently, (156) and (161) - (163) hold, with $\dot{g}_{BA}^s > 0$, $\dot{g}_{BB}^s < 0$ and $\dot{b}_B^d > 0$.

For $t \in [\tilde{t}_c, \chi)$, we get (153) - (155), so that (156) - (158) hold. Because of Lemma 4, $\chi \neq t_c$. Because $g_{BB}^s(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because $\dot{g}_{BA}^s > 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = T_A$.

For $t \in [T_A, \chi)$, we get (164), (154) and (155), so that (173) and

$$\dot{b}_B^d = \frac{\rho\lambda}{U''} < 0, \quad \dot{g}_{BA}^s = 0 \quad (185)$$

hold. Because $g_{BB}^s(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because $\dot{g}_{BA}^s = 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) = 0$, $\chi \neq T_A$. Because $\dot{p}_A < \rho\lambda^{\text{st}}$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) > 0$, $\chi \neq \tilde{T}_B$. Thus, $\chi = t_c$.

For $t \in [t_c, \chi)$, we get (164), (159) and (160), so that (165) - (169) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because of Lemma 4, $\chi \neq \tilde{t}_c$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) = 0$, $\chi \neq T_A$. Because $b_B^d(t) > 0$, $\chi \neq \tilde{T}_B$. Thus, $\chi = \{t_d, \tilde{T}_A, T_B\}$.

Case $[0, \tilde{t}_c), [\tilde{t}_c, T_A), [T_A, t_c), [t_c, \tilde{T}_A)$ Suppose that $0 < \tilde{t}_c < T_A < t_c < \tilde{T}_A < \dots$. Then, for $t \in [\tilde{T}_A, \chi)$ we get (153), (159) and (160), so that (156) and (161) - (163) with $\dot{g}_{BA}^s > 0$ hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $\dot{g}_{BA}^s > 0$, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because of Lemma 4, $\chi \neq T_A$. If $\chi = \tilde{t}_c^2$, the equation system describing the market equilibrium is identical to the one for $t = \tilde{t}_c^1$. Consequently, the proof of Lemma 4 implies $\chi \neq \tilde{t}_c^2$. Thus, $\chi = T_B$.

For $t \in [T_B, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Following the related argument yields the sequence $0 < \tilde{t}_c < T_A^1 < t_c < \tilde{T}_A < T_B < T_A^2$.

Case $[0, \tilde{t}_c), [\tilde{t}_c, T_A), [T_A, t_c), [t_c, T_B)$ Suppose that $0 < \tilde{t}_c < T_A < t_c < T_B < \dots$. Then, for $t \in [T_B, \chi)$ we get (164), (170) and (160). Because the solution of this equation system does not depend on time, we get the sequence $0 < \tilde{t}_c < T_A < t_c < T_B$.

Case $[0, \tilde{t}_c), [\tilde{t}_c, T_A), [T_A, t_c), [t_c, t_d)$ Suppose that $0 < \tilde{t}_c < T_A < t_c < t_d < \dots$. Then, for $t \in [t_d, \chi]$ we get (171) and (172), so that (173) and (174) hold. Following the related argument yields the sequence $0 < \tilde{t}_c < T_A < t_c < t_d < T_B$.

B.1.2 SE < 0 and $g_{BA}^s(0) > 0$

Suppose now that the strategic effect is negative, so that $\rho\lambda > \rho\lambda^{\text{ST}}$, but not strong enough to rule out green energy exports from B to A initially. Similar to section B.1.1, the assumptions $b_A(0)^d, b_B^d(0), g_{BA}^s(0) > 0$ rule out $\tilde{T}_A, \tilde{T}_B, \tilde{t}_d, t_e, \tilde{t}_e, t_f$ and $\tilde{t}_f$ as first element of the sequence.

Case $[0, t_c)$

For $t \in [0, t_c)$, we get (153) - (155), so that (156) - (158) hold. For $t \in [t_c, \chi)$, we get (153), (159) and (160), so that (156) and (161) - (163) hold with $\dot{g}_{BA}^s < 0$, $\dot{g}_{BB}^s > 0$, $\dot{b}_A^d < 0$ and $\dot{b}_B^d < 0$. Because the argument made in section B.1.1 holds, $\chi = \{t_d, T_A, T_B\}$.

Case $[0, t_c), [t_c, T_A]$ Suppose that $0 < t_c < T_A < \dots$. The arguments made in section B.1.1 hold implying the sequences $0 < t_c < T_A < T_B$ and $0 < t_c < T_A < t_d < T_B$.

Case $[0, t_c), [t_c, T_B]$ Suppose that $0 < t_c < T_B < \dots$. The arguments made in section B.1.1 hold implying the sequences $0 < t_c < T_B < T_A$.

Case $[0, t_c), [t_c, t_d]$ Suppose that $0 < t_c < t_d < \dots$. Then, for $t \in [t_d, \chi)$ we get (180) and (172), so that (156), (174) and (181) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because of Lemma 4, $\chi \neq \tilde{t}_d$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = \{T_A, T_B, \tilde{t}_e\}$.

Case $[0, t_c), [t_c, t_d), [t_d, T_A]$ Suppose that $0 < t_c < t_d < T_A < \dots$. The arguments made in section B.1.1 hold implying the sequence $0 < t_c < t_d < T_A < T_B$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B]$ Suppose that $0 < t_c < t_d < T_B < \dots$. For $t \in [T_B, \chi)$, the argument made in section B.1.1 hold implying $\chi = \{\tilde{t}_d, T_A\}$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B), [T_B, T_A]$ Suppose that $0 < t_c < t_d < T_B < T_A < \dots$. The argument made in section B.1.1 hold implying the sequence $0 < t_c < t_d < T_B < T_A$.

Case $[0, t_c), [t_c, t_d), [t_d, T_B), [T_B, \tilde{t}_d)$ Suppose that $0 < t_c < t_d < T_B < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $b_B^d(t) = 0$, $\chi \neq \{\tilde{t}_c, T_B\}$. Because of Lemma 4, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $\dot{p}_B < \rho\lambda$, $\chi \neq \tilde{T}_B$. Thus, $\chi = T_A$.

For $t \in [T_A, \chi)$, we get (164), (170) and (160). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_c < t_d < T_B < \tilde{t}_d < T_A$.

Case $[0, t_c), [t_c, t_d), [t_d, \tilde{t}_e)$ Suppose that $0 < t_c < t_d < \tilde{t}_e < \dots$. For $t \in [\tilde{t}_e, \chi)$, we get

$$p_A(t) = M'_A(g_{AA}^s(t) + g_{AB}^s(t)) = U'(b_A^d(t) + g_{AA}^s(t)) = c + \tau^u(t) + \lambda^{\text{ST}}(t), \quad (186)$$

$$p_B(t) = M'_B(g_{BB}^s(t)) = U'(b_B^d(t) + g_{BB}^s(t) + \alpha g_{AB}^s(t)) = \rho + \lambda(t), \quad (187)$$

$$\alpha p_B(t) = p_A(t) + Q'(g_{AB}^s(t)). \quad (188)$$

Differentiating with respect to time yields (156) and

$$\dot{g}_{AA}^s = \frac{\rho\lambda^{\text{ST}}}{M_A''} - \dot{g}_{AB}^s \leq 0, \quad \dot{b}_A^d = \frac{\rho\lambda^u}{U''} - \dot{g}_{AA}^s \leq 0, \quad (189)$$

$$\dot{g}_{BB}^s = \frac{\rho\lambda}{M_B''} > 0, \quad (190)$$

$$\dot{g}_{AB}^s = \frac{\alpha\rho\lambda - \rho\lambda^{\text{ST}}}{Q''}, \quad \dot{b}_B^d = \frac{\rho\lambda}{U''} - \dot{g}_{BB}^s - \alpha\dot{g}_{AB}^s. \quad (191)$$

Before $t = \tilde{t}_e$, $\alpha p_B(t) < p_A(t) + Q'(0)$ holds. Consequently, $\tilde{t}_e$ can be only part of the sequence if $\alpha\dot{p}_B > \dot{p}_A \Leftrightarrow \alpha\rho\lambda - \rho\lambda^{\text{ST}} > 0$, which implies $\dot{g}_{AB}^s > 0$ and $\dot{b}_B^d < 0$.

Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because of Lemma 4, $\chi \neq t_e$. Because $g_{AA}^s(t) > 0$, $\chi \neq t_f$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Thus, $\chi = \{\tilde{t}_f, T_B\}$.

Case $[0, t_c), [t_c, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f)$ Suppose that $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < \dots$. For $t \in [\tilde{t}_f, \chi)$, we get (187) and

$$p_A(t) = U'(b_A^d(t)) = c + \tau^u(t) + \lambda^{\text{ST}}(t), \quad (192)$$

$$\alpha p_B(t) = M'_A(g_{AB}^s(t)) + Q'(g_{AB}^s(t)). \quad (193)$$

Differentiating yields (156) and

$$\dot{g}_{AB}^s = \frac{\alpha\rho\lambda}{M_A'' + Q''} > 0, \quad \dot{b}_A^d = \frac{\rho\lambda^{\text{ST}}}{U''} < 0, \quad (194)$$

$$\dot{g}_{BB}^s = \frac{\rho\lambda}{M_B''} > 0, \quad \dot{b}_B^d = \frac{\rho\lambda}{U''} - \dot{g}_{BB}^s - \alpha\dot{g}_{AB}^s < 0. \quad (195)$$

Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $\dot{g}_{AB}^s > 0$, $\chi \neq t_e$. Because of Lemma 4, $\chi \neq t_f$. Because $g_{AA}^s(t) = 0$, $\chi \neq \{T_A, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = T_B$.

For $t \in [T_B, \chi)$, we get (192), (193) and

$$p_B(t) = U'(g_{BB}^s(t) + \alpha g_{AB}^s) = M'_B(g_{BB}^s(t)). \quad (196)$$

Differentiating with yields (182) and

$$\dot{b}_A^d = \frac{\rho \lambda^{\text{ST}}}{U''} < 0, \quad \dot{g}_{BB}^s = \dot{g}_{AB}^s = 0. \quad (197)$$

Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $\dot{g}_{AB}^s = 0$, $\chi \neq t_e$. Because $g_{AA}^s(t) = 0$, $\chi \neq \{T_A, \tilde{t}_f\}$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Because of Lemma 4, $\chi \neq \tilde{T}_B$. Thus, $\chi = t_f$.

For $t \in [t_f, \chi)$, we get (186), (196) and (188). Differentiating yields

$$\dot{p}_A = \rho \lambda^{\text{ST}} > 0, \quad \dot{p}_B = \frac{\alpha \rho \lambda^{\text{ST}} U'' M_B''}{\alpha^2 U'' M_B'' - Q''[M_B'' - U'']} > 0, \quad (198)$$

$$\dot{g}_{AA}^s = \frac{\rho \lambda^{\text{ST}}}{M_A''} - \dot{g}_{AB}^s > 0, \quad \dot{g}_{AB}^s = \frac{\rho \lambda^{\text{ST}} [M_B'' - U'']}{\alpha^2 U'' M_B'' - Q''[M_B'' - U'']} < 0, \quad (199)$$

$$\dot{g}_{BB}^s = \frac{\alpha \rho \lambda^{\text{ST}} U''}{\alpha^2 U'' M_B'' - Q''[M_B'' - U'']} > 0, \quad (200)$$

$$\dot{b}_A^d = \frac{\rho \lambda^{\text{ST}}}{U''} - \dot{g}_{AA}^s < 0. \quad (201)$$

Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AA}^s > 0$, $\chi \neq t_f$. Because of Lemma 4, $\chi \neq \tilde{t}_f$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$.

Suppose that $\chi = \tilde{T}_B$. Then, for $t \in [\tilde{T}_B, \chi)$ we get (186) - (188), so that (189) - (191) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $\dot{g}_{AB}^s > 0$, $\chi \neq t_e$. Because $g_{AA}^s(t) > 0$, $\chi \neq t_f$. In case of $\chi = \tilde{t}_f^2$, the equation system is identical to the one at $t = \tilde{t}_f^1$, so that the proof of Lemma 4 implies $\chi \neq \tilde{t}_f^2$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Finally, Lemma 4 implies $\chi \neq T_B$. As there is no element that can follow $\tilde{T}_B$, $\tilde{T}_B$ cannot follow t_f in case of $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < \dots$ implying $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \dots$

For $t \in [t_e, \chi)$, we get (180) and (175), so that (181) - (183) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because of Lemma 4, $\chi \neq \tilde{t}_e$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Because $\dot{p}_B < \rho \lambda$, $\chi \neq \tilde{T}_B$. Thus, $\chi = \{T_A, \tilde{t}_d\}$.

Case $[0, t_c), [t_c, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f), [\tilde{t}_f, T_B), [T_B, t_f), [t_f, t_e), [t_e, T_A)$ Suppose that $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A < \dots$. For $t > T_A$, we get (171) and (175). Because the solution of this equation system does not depend on time, we get the sequence $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A$.

Case $[0, t_c), [t_c, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f), [\tilde{t}_f, T_B), [T_B, t_f), [t_f, t_e), [t_e, \tilde{t}_d)$ Suppose that $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. By using the related argument and by taking into account that $\dot{p}_B < \rho\lambda$ implies $\chi \neq \tilde{T}_B$ we get the sequence $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A$.

Case $[0, t_c), [t_c, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, T_B)$ Suppose that $0 < t_c < t_d < \tilde{t}_e < T_B < \dots$. For $t \in [T_B, \chi)$, we get (186), (196) and (188), so that (198) - (201) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AA}^s(t) > 0$, $\chi \neq t_f$. Because $\dot{g}_{AA}^s > 0$ and $\dot{g}_{AB}^s < 0$, $\chi \neq \tilde{t}_f$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Because of Lemma 4, $\chi \neq \tilde{T}_B$. Thus, $\chi = t_e$.

For $t \in [t_e, \chi)$, we get (180) and (175), so that (181) - (183) hold. Following the related argument yields $\chi = \{\tilde{t}_d, T_A\}$.

Case $[0, t_c), [t_c, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, T_B), [T_B, t_e), [t_e, T_A)$ Suppose that $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < T_A < \dots$. For $t > T_A$, we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < T_A$.

Case $[0, t_c), [t_c, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, T_B), [T_B, t_e), [t_e, \tilde{t}_d)$ Suppose that $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and that $\dot{p}_B < \rho\lambda$ implies $\chi \neq \tilde{T}_B$ yields the sequence $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A$.

Case $[0, t_d)$

For $t \in [0, t_d)$, we get (153), (159) and (160), so that the dynamics are given by (156) and (161) - (163), with $\dot{g}_{BA}^s < 0$, $\dot{g}_{BB}^s > 0$ and $\dot{b}_B^d < 0$.

For $t \in [t_d, \chi)$, we get (180) and (172), so that (156), (174) and (181) hold. Because $g_{BA}^s(t) = g_{AB}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d, t_e, t_f, \tilde{t}_f\}$. Because of Lemma 4, $\chi \neq \tilde{t}_f$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = \{T_A, T_B, \tilde{t}_e\}$.

Case $[0, t_d), [t_d, T_A)$ Suppose that $0 < t_d < T_A < \dots$. For $t \in [T_A, \chi)$, we get (171) and (172), so that (173) and (174) hold. Because of the related argument, we get the sequence $0 < t_d < T_A < T_B$.

Case $[0, t_d), [t_d, T_B)$ Suppose that $0 < t_d < T_B < \dots$. For $t \in [T_B, \chi)$, we get (180) and (175), so that (181) - (183) hold. The related argument yields $\chi = \{T_A, \tilde{t}_d\}$.

Case $[0, t_d), [t_d, T_B), [T_B, T_A)$ Suppose that $0 < t_d < T_B < T_A < \dots$. For $t > T_A$, we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_d < T_B < T_A$.

Case $[0, t_d), [t_d, T_B), [T_B, \tilde{t}_d)$ Suppose that $0 < t_d < T_B < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi]$, we get (153), (170) and (160), so that (176) - (179) hold. Taking the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ into account yields the sequence $0 < t_d < T_B < \tilde{t}_d < T_A$.

Case $[0, t_d), [t_d, \tilde{t}_e)$ Suppose that $0 < t_d < \tilde{t}_e < \dots$. For $t \in [\tilde{t}_e, \chi]$, we get (186) - (188), so that (156) and (189) - (191) hold. The related argument yields $\chi = \{T_B, \tilde{t}_f\}$.

Case $[0, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, T_B)$ Suppose that $0 < t_d < \tilde{t}_e < T_B < \dots$. For $t \in [T_B, \chi]$, we get (186), (196) and (188), so that (198) - (201) hold. Using the arguments made for the cases $0 < t_c < t_d < \tilde{t}_e < T_B < \dots$, $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < T_A < \dots$ and $0 < t_c < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d \dots$ yield the sequences $0 < t_d < \tilde{t}_e < T_B < t_e < T_A$ and $0 < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A$.

Case $[0, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f)$ Suppose that $0 < t_d < \tilde{t}_e < \tilde{t}_f < \dots$. For $t \in [\tilde{t}_f, \chi]$, we get (192), (187) and (193), so that (156), (194) and (195) hold. Following the argument made for the case $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < \dots$ leads to $0 < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \dots$, with $\chi = \{T_A, \tilde{t}_d\}$ as next element. Because the arguments made for the cases $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A < \dots$ and $0 < t_c < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < \dots$ hold, we get the sequences $0 < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A$ and $0 < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A$.

Case $[0, T_A)$, with $g_{BB}^s(0) > 0$

For $t \in [0, T_A)$, we get (153), (159), (160), so that (156) and (161) - (163) hold, with $\dot{g}_{BA}^s < 0$, $\dot{g}_{BB}^s > 0$, $\dot{b}_B^d < 0$.

For $t \in [T_A, \chi)$, we get (164), (159), (160), so that (165) - (169) hold. Using the related argument yields $\chi = \{t_d, T_B\}$.

Case $[0, T_A), [T_A, T_B)$ Suppose that $0 < T_A < T_B < \dots$. For $t > T_B$, we get (164), (170) and (160). Because the solution of the equation system does not depend on time, we get the sequence $0 < T_A < T_B$.

Case $[0, T_A), [T_A, t_d)$ Suppose that $0 < T_A < t_d < \dots$. For $t \in [t_d, \chi)$, we get (171) and (172), so that (173) and (174) hold. Following the related argument yields the sequence $0 < T_A < t_d < T_B$.

Case $[0, T_A)$, with $g_{BB}^s(0) = 0$

For $t \in [0, T_A)$, we get (153) - (155), so that (156) - (158) hold.

For $t \in [T_A, \chi)$, we get (164), (154) and (155), so that (173) and (184) hold. Following the related argument yields $\chi = \{t_c, \tilde{t}_d\}$.

Case $[0, T_A), [T_A, t_c)$ Suppose that $0 < T_A < t_c < \dots$. For $t \in [t_c, \chi)$, we get (164), (159) and (160), so that (165) - (169) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because of Lemma 4, $\chi \neq \tilde{t}_c$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t) = 0$, $\chi \neq T_A$. Because $b_B^d(t) > 0$, $\chi \neq \tilde{T}_B$. Thus, $\chi = \{\tilde{T}_A, T_B, t_d\}$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < \dots$. For $t \in [\tilde{T}_A, \chi)$, we get (153), (159) (160), so that (156) and (161) - (163) hold, with $\dot{g}_{BA} < 0$, $\dot{g}_{BB}^s > 0$ and $\dot{b}_B^d < 0$. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $\dot{g}_{BA}^s < 0$ and $\dot{g}_{BB}^s > 0$, $\chi \neq \tilde{t}_c$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because of Lemma 4, $\chi \neq T_A$. Thus, $\chi = \{t_d, T_B\}$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < \dots$. For $t \in [t_d, \chi)$, we get (180) and (172), so that (156), (174) and (181) hold. Because $g_{BA}^s(t) = g_{AB}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d, t_e, t_f, \tilde{t}_f\}$. Because $\alpha \dot{p}_A - \dot{p}_B < 0$, $\chi \neq \tilde{t}_d$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = \{T_A, T_B, \tilde{t}_e\}$.

Case $[0, T_A^1), [T_A^1, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_A^2)$ Suppose that $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_A^2 < \dots$. For $t \in [T_A^2, \chi)$, we get (171) and (172), so that (173) and (174) hold. Following the related argument yields the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_A^2 < T_B$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_B)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < T_B < \dots$. For $t \in [T_B, \chi)$, we get (180) and (175), so that (181) - (183) hold. Following the related argument yields $\chi = \{\tilde{t}_d, T_A\}$.

Case $[0, T_A^1), [T_A^1, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_B), [T_B, T_A^2)$ Suppose that $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < T_A^2 < \dots$. For $t > T_A^2$, we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, T_B), [T_B, \tilde{t}_d)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < T_B < \tilde{t}_d < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, \tilde{t}_e)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \dots$. For $t \in [\tilde{t}_e, \chi)$, we get (186) - (188), so that (156) and (189) - (191) hold. Because $\alpha p_B(t) < p_A(t) + Q'(0)$ before $t = \tilde{t}_e$, $\dot{g}_{AB}^s > 0$ and $\dot{b}_B^d < 0$. Following the related argument yields $\chi = \{T_B, \tilde{t}_f\}$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, T_B)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < \tilde{t}_e < T_B < \dots$. For $t \in [T_B, \chi)$, we get (186), (196) and (188), so that (198) - (201) hold. Taking account of Lemma 4 and the related argument implies that t_e follows T_B and that $\chi = \{T_A, \tilde{t}_d\}$ holds for $t \in [t_e, \chi)$.

Case $[0, T_A^1), [T_A^1, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, T_B), [T_B, t_e), [t_e, T_A^2)$ Suppose that $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < T_B < t_e < T_A^2 < \dots$. For $t > T_A^2$ we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < T_B < t_e < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, T_B), [T_B, t_e), [t_e, \tilde{t}_d)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < \dots$. For $t \in [\tilde{t}_f, \chi)$, we get (192), (187) and (193), so that (156), (194) and (195) hold. Following the related arguments yields $0 < T_A < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \dots$, with either $\tilde{t}_d$ or T_A as next element.

Case $[0, T_A^1), [T_A^1, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f), [\tilde{t}_f, T_B), [T_B, t_f), [t_f, t_e), [t_e, T_A^2)$ Suppose that $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A^2 < \dots$. For $t > T_A^2$, we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, t_d), [t_d, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f), [\tilde{t}_f, T_B), [T_B, t_f), [t_f, t_e), [t_e, \tilde{t}_d)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < T_A^1 < t_c < \tilde{T}_A < t_d < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, \tilde{T}_A), [\tilde{T}_A, T_B)$ Suppose that $0 < T_A < t_c < \tilde{T}_A < T_B < \dots$. For $t \in [T_B, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < T_A^1 < t_c < \tilde{T}_A < T_B < T_A^2$.

Case $[0, T_A), [T_A, t_c), [t_c, T_B)$ Suppose that $0 < T_A < t_c < T_B < \dots$. For $t > T_B$, we get (164), (170) and (160). Because the solution of the equation system does not depend on time, we get the sequence $0 < T_A < t_c < T_B$.

Case $[0, T_A), [T_A, t_c), [t_c, t_d)$ Suppose that $0 < T_A < t_c < t_d < \dots$. For $t_d \in [\chi)$, we get (171) and (172), so that (173) and (174) hold. Following related argument yields the sequence $0 < T_A < t_c < t_d < T_B$.

Case $[0, T_A), [T_A, t_d)$ Suppose that $0 < T_A < t_d < \dots$. For $t \in [t_d, \chi)$, we get (171) and (172), so that (173) and (174) hold. Following related argument yields the sequence $0 < T_A < t_d < T_B$.

Case $[0, T_B)$

For $t \in [0, T_B)$, we get (153), (159) and (160), so that (156) and (161) - (163) hold, with $\dot{g}_{BA}^s < 0$, $\dot{g}_{BB}^s > 0$ and $\dot{b}_B^d < 0$.

For $t \in [T_B, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < T_B < T_A$.

B.1.3 SE < 0 and $g_{AB}^s(0) > 0$

Suppose that the strategic effect is negative and sufficiently strong to render green energy exports from country A to country B profitable. The assumptions $b_A^d(0), b_B^d(0), g_{AB}^s(0) > 0$ rule out $\tilde{T}_A, \tilde{T}_B, t_c, \tilde{t}_c, t_d, \tilde{t}_d$ and $\tilde{t}_e$ as first element of the sequence. Because $\bar{p}_A > \bar{p}_B$, T_A can also not be the first element.

Case $[0, t_e)$

For $t \in [0, t_e)$, we get (186) - (188), so that (156) and (189) - (191) hold, with $\dot{g}_{AB}^s < 0$, $\dot{g}_{AA}^s > 0$ and $\dot{b}_A^d < 0$.

For $t \in [t_e, \chi)$, we get (180) and (172), so that (156), (174) and (181) hold. Taking account of the related argument and $\alpha \dot{p}_A - \dot{p}_B < 0 \Rightarrow \chi \neq \tilde{t}_d$ yields $\chi = \{T_A, T_B\}$.

Case $[0, t_e), [t_e, T_A)$ Suppose that $0 < t_e < T_A < \dots$. For $t \in [T_A, \chi)$, we get (171), (172), so that (173) and (174) hold. Following the related argument yields the sequence $0 < t_e < T_A < T_B$.

Case $[0, t_e), [t_e, T_B)$ Suppose that $0 < t_e < T_B < \dots$. For $t \in [T_B, \chi)$, we get (180) and (175), so that (181) - (183) hold. Following the related argument yields $\chi = \{T_A, \tilde{t}_d\}$.

Case $[0, t_e), [t_e, T_B), [T_B, T_A)$ Suppose that $0 < t_e < T_B < T_A < \dots$. For $t > T_A$, we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_e < T_B < T_A$.

Case $[0, t_e), [t_e, T_B), [T_B, \tilde{t}_d)$ Suppose that $0 < t_e < T_B < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < t_e < T_B < \tilde{t}_d < T_A$.

Case $[0, t_f]$

For $t \in [0, t_f]$, we get (192), (187) and (193), so that (156), (194) and (195) hold.

For $t \in [t_f, \chi)$, we get (186) - (188), so that (156) and (189) - (191) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AA}^s(t) > 0$, $\chi \neq t_f$. Because of Lemma 4, $\chi \neq \tilde{t}_f$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Thus, $\chi = \{t_e, T_B\}$.

Case $[0, t_f), [t_f, t_e)$ Suppose that $0 < t_f < t_e < \dots$. For $t \in [t_e, \chi)$, we get (180), (172), so that (156), (174) and (181) hold. Using the arguments from Case $[0, t_e)$ of this subsection yields the sequences $0 < t_f < t_e < T_A < T_B$, $0 < t_f < t_e < T_B < T_A$ and $0 < t_f < t_e < T_B < \tilde{t}_d < T_A$.

Case $[0, t_f), [t_f, T_B)$ Suppose that $0 < t_f < T_B < \dots$. For $t \in [T_B, \chi)$, we get (186), (196) and (188), so that (198) - (201) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AA}^s(t) > 0$, $\chi \neq t_f$. Because $\dot{g}_{AA}^s > 0 > \dot{g}_{AB}^s$, $\chi \neq \tilde{t}_f$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because of Lemma 4, $\chi \neq \tilde{T}_B$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Thus, $\chi = t_e$.

For $t \in [t_e, \chi)$, we get (180) and (175), so that (181) - (183) hold. Following the related argument yields $\chi = \{T_A, \tilde{t}_d\}$.

Case $[0, t_f), [t_f, T_B), [T_B, t_e), [t_e, T_A)$ Suppose that $0 < t_f < T_B < t_e < T_A < \dots$. For $t > T_A$, we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < t_f < T_B < t_e < T_A$.

Case $[0, t_f), [t_f, T_B), [T_B, t_e), [t_e, \tilde{t}_d)$ Suppose that $0 < t_f < T_B < t_e < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < t_f < T_B < t_e < \tilde{t}_d < T_A$.

Case $[\tilde{t}_f, 0)$

For $t \in [0, \tilde{t}_f)$ we get (186) - (188), so that (156) and (189) - (191) hold, with $\dot{g}_{AB}^s > 0$ and $\dot{b}_B^d < 0$.

For $t \in [\tilde{t}_f, \chi)$, we get (192), (187) and (193), so that (156), (194) and (195) hold. Following the related arguments yields $0 < \tilde{t}_f < T_B < t_f < t_e < \dots$ with either $\tilde{t}_d$ or T_A as the next element.

Case $[0, \tilde{t}_f), [\tilde{t}_f, T_B), [T_B, t_f), [t_f, t_e), [t_e, T_A)$ Suppose that $0 < \tilde{t}_f < T_B < t_f < t_e < T_A < \dots$. For $t > T_A$, we get (171) and (175). Because the solution of the equation system does not depend on time, we get the sequence $0 < \tilde{t}_f < T_B < t_f < t_e < T_A$.

Case $[0, \tilde{t}_f), [\tilde{t}_f, T_B), [T_B, t_f), [t_f, t_e), [t_e, \tilde{t}_d)$ Suppose that $0 < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < \dots$. For $t \in [\tilde{t}_d, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Taking account of

the related argument and $\dot{p}_B < \rho\lambda \Rightarrow \chi \neq \tilde{T}_B$ yields the sequence $0 < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A$.

Case $[0, T_B)$, with $g_{AA}^s(0) > 0$

For $t \in [0, T_B)$, we get (186) - (188), so that (156) and (189) - (191) hold, with $\dot{b}_B^d < 0$.

For $t \in [T_B, \chi)$, we get (186), (196) and (188), so that (198) - (201) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AA}^s(t) > 0$, $\chi \neq t_f$. Because $\dot{g}_{AA}^s > 0 > \dot{g}_{AB}^s$, $\chi \neq \tilde{t}_f$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Because of Lemma 4, $\chi \neq \tilde{T}_B$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Thus, $\chi = t_e$.

For $t \in [t_e, \chi)$, we get (180) and (175), so that (181) - (183) hold. Following the related arguments yields $\chi = \{\tilde{t}_d, T_A\}$. Subsequently, the arguments used for the cases $0 < t_e < T_B < T_A < \dots$ and $0 < t_e < T_B < \tilde{t}_d < \dots$ hold implying the sequences $0 < T_B < t_e < T_A$ and $0 < T_B < t_e < \tilde{t}_d < T_A$.

Case $[0, T_B)$, with $g_{AA}^s(0) = 0$

For $t \in [0, T_B)$, we get (192), (187) and (193), so that (156), (194) and (195) hold.

For $t \in [T_B, \chi)$, we get (192), (196) and (193), so that (182) and (197) hold. Following the related argument yields $\chi = t_f$.

For $t \in [t_f, \chi)$, we get (186), (196) and (188), so that (198) - (201) hold. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AA}^s(t) > 0$, $\chi \neq t_f$. Because of Lemma 4, $\chi \neq \tilde{t}_f$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Because $b_A^d(t) > 0$, $\chi \neq \tilde{T}_A$. Because $b_B^d(t) = 0$, $\chi \neq T_B$. Thus, $\chi = \{t_e, \tilde{T}_B\}$.

Case $[0, T_B), [T_B, t_f), [t_f, t_e)$ Suppose that $0 < T_B < t_f < t_e < \dots$. For $t \in [t_e, \chi)$, we get (180) and (175), so that (181) - (183) hold. Following the related arguments yields $\chi = \{\tilde{t}_d, T_A\}$. Subsequently, the arguments used for the cases $0 < t_e < T_B < T_A < \dots$ and $0 < t_e < T_B < \tilde{t}_d < \dots$ hold implying the sequences $0 < T_B < t_f < t_e < T_A$ and $0 < T_B < t_f < t_e < \tilde{t}_d < T_A$.

Case $[0, T_B), [T_B, t_f), [t_f, \tilde{T}_B)$ Suppose that $0 < T_B < t_f < \tilde{T}_B < \dots$. For $t \in [\tilde{T}_B, \chi)$, we get (186) - (188), so that (156) and (189) - (191) hold, with $\dot{b}_B^d > 0$, $\dot{g}_{AB}^s < 0$, $\dot{g}_{AA}^s > 0$ and $\dot{b}_A^d < 0$. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because of Lemma 4, $\chi \neq T_B$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Thus, $\chi = t_e$.

For $t \in [t_e, \chi)$, we get (180) and (172), so that (156), (174) and (181) hold. Taking account of the related argument and $\alpha \dot{p}_A - \dot{p}_B < 0 \Rightarrow \chi \neq \tilde{t}_d$ yields $\chi = \{T_A, T_B\}$.

Case $[0, T_B), [T_B, t_f), [t_f, \tilde{T}_B), [\tilde{T}_B, t_e), [t_e, T_A)$ Suppose that $0 < T_B < t_f < \tilde{T}_B < t_e < T_A < \dots$. For $t \in [T_A, \chi)$, we get (171) and (172), so that (173) and (174) hold. The related argument yields the sequence $0 < T_B^1 < t_f < \tilde{T}_B < t_e < T_A < T_B^2$.

Case $[0, T_B^1), [T_B^1, t_f), [t_f, \tilde{T}_B), [\tilde{T}_B, t_e), [t_e, T_B^2)$ Suppose that $0 < T_B^1 < t_f < \tilde{T}_B < t_e < T_B^2 < \dots$. For $t \in [T_B^2, \chi)$, we get (180) and (175), so that (181) - (183) hold. Following the related arguments yields $\chi = \{\tilde{t}_d, T_A\}$. Subsequently, the arguments used for the cases $0 < t_e < T_B < T_A < \dots$ and $0 < t_e < T_B < \tilde{t}_d < \dots$ hold implying the sequences $0 < T_B^1 < t_f < \tilde{T}_B < t_e < T_B^2 < T_A$ and $0 < T_B^1 < t_f < \tilde{T}_B < t_e < T_B^2 < \tilde{t}_d < T_A$.

□

B.2 Timings without initial trade

Lemma 7 *Suppose that green energy is not traded initially, fossil fuel is consumed in both countries initially and country A acts strategically. In the unilaterally regulated economy with the fossil fuel tax τ^{ST} in country A and no fuel tax in country B the timing is given by*

- i) $0 < \tilde{t}_d < \tilde{t}_c < T_A^1 < t_c < \tilde{T}_A < T_B < T_A^2$, viii) $0 < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A$,
- ii) $0 < \tilde{t}_d < \tilde{t}_c < T_A < t_c < T_B$, ix) $0 < \tilde{t}_e < T_B < t_e < T_A$,
- iii) $0 < \tilde{t}_d < \tilde{t}_c < T_A < t_c < t_d < T_B$, x) $0 < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A$,
- iv) $0 < \tilde{t}_d < T_A < T_B$, xi) $0 < T_B < \tilde{t}_d < T_A$,
- v) $0 < \tilde{t}_d < T_A < t_d < T_B$, xii) $0 < T_B^1 < \tilde{t}_d < \tilde{T}_B < T_A < T_B^2$,
- vi) $0 < \tilde{t}_d < T_B < T_A$, xiii) $0 < T_B^1 < \tilde{t}_d < \tilde{T}_B < \tilde{t}_c < T_A < t_c < T_B^2$.
- vii) $0 < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A$,

Proof Suppose that $g_{BA}^s(t) = g_{AB}^s(t) = 0$ at least until the first element of the sequence. Then, $t_c, \tilde{t}_c, t_d, t_e, t_f$ and $\tilde{t}_f$ cannot be the first element, because there is no trade. Because $b_A(t), b_B(t) > 0$, neither $\tilde{T}_A$ nor $\tilde{T}_B$ can be the first element. If T_A is the first element, $\bar{p}_A > \bar{p}_B$ rules out trade for all points in time. Thus, the first element is $\tilde{t}_d, \tilde{t}_e$ or T_B . In all three cases, the market equilibrium before the first element of the sequence is described by (180) and (172) implying that (156), (174) and (181) describe the evolution over time.

Case $[0, \tilde{t}_d)$

Suppose that $\tilde{t}_d$ is the first element. Then, $\alpha \dot{p}_A > \dot{p}_B$ has to hold for $t \in [0, \tilde{t}_d)$ implying $SE > 0$. For $t \in [\tilde{t}_d, \chi)$ we get (153), (159) and (160) implying that (156) and (161) - (163) hold with $\dot{g}_{BA}^s > 0$ and $\dot{b}_A^d < 0$. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because of Lemma 4, $\chi \neq t_d$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Thus, $\chi = \{\tilde{t}_c, T_A, T_B\}$.

Case $[0, \tilde{t}_d), [\tilde{t}_d, \tilde{t}_c)$ Suppose that $0 < \tilde{t}_d < \tilde{t}_c < \dots$. Then, for $t \in [\tilde{t}_c, \chi)$, we get (153) - (155) implying that (156) - (158) hold. Applying the argument of Appendix B.1.1, **Case** $[0, \tilde{t}_c]$ yields the timings $0 < \tilde{t}_d < \tilde{t}_c < T_A^1 < t_c < \tilde{T}_A < T_B < T_A^2$, $0 < \tilde{t}_d < \tilde{t}_c < T_A < t_c < T_B$ and $0 < \tilde{t}_d < \tilde{t}_c < T_A < t_c < t_d < T_B$.

Case $[0, \tilde{t}_d), [\tilde{t}_d, t_A)$ Suppose that $0 < \tilde{t}_d < T_A < \dots$. Then, for $t \in [T_A, \chi)$, we get (164), (159) and (160), so that (165) - (169) hold. Applying the argument of Appendix B.1.1, **Case** $[0, T_A)$, **with** $g_{BB}^s(0) > 0$ yields the sequences $0 < \tilde{t}_d < T_A < T_B$ and $0 < \tilde{t}_d < T_A < t_d < T_B$.

Case $[0, \tilde{t}_d), [\tilde{t}_d, T_B)$ Suppose that $0 < \tilde{t}_d < T_B < \dots$. Then, for $t \in [T_B, \chi)$, we get (153), (170) and (160), so that (176) - (179) hold. Following the related argument yields the timing $0 < \tilde{t}_d < T_B < T_A$.

Case $[0, \tilde{t}_e)$

Suppose that $\tilde{t}_e$ is the first element. Then, $\alpha \dot{p}_B > \dot{p}_A$ has to hold for $t \in [0, \tilde{t}_e)$ implying $SE < 0$. For $t \in [\tilde{t}_e, \chi)$ we get (186) - (188) implying that (156) and (189) - (191) hold with $\dot{g}_{AB}^s > 0$ and $\dot{b}_B^d < 0$. Because $g_{BA}^s(t) = 0$, $\chi \neq \{t_c, \tilde{t}_c, t_d\}$. Because $g_{AB}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because of Lemma 4, $\chi \neq t_e$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because $\bar{p}_A > \bar{p}_B$, $\chi \neq T_A$. Thus, $\chi = \{\tilde{t}_f, T_B\}$.

Case $[0, \tilde{t}_e), [\tilde{t}_e, \tilde{t}_f)$ Suppose that $0 < \tilde{t}_e < \tilde{t}_f < \dots$. Then, for $t \in [\tilde{t}_f, \chi)$, we get (192), (187) and (193), so that (156), (194) and (195) hold. Applying the argument of Appendix B.1.3, **Case** $[0, \tilde{t}_f)$ yields the timings $0 < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < T_A$ and $0 < \tilde{t}_e < \tilde{t}_f < T_B < t_f < t_e < \tilde{t}_d < T_A$.

Case $[0, \tilde{t}_e), [\tilde{t}_e, T_B)$ Suppose that $0 < \tilde{t}_e < T_B < \dots$. Then, for $t \in [T_B, \chi)$, we get (186), (196) and (188), so that (198) - (201) hold. Applying the argument of Appendix B.1.3, **Case** $[0, T_B)$, **with** $g_{AA}^s(0) > 0$ yields the timings $0 < \tilde{t}_e < T_B < t_e < T_A$ and $0 < \tilde{t}_e < T_B < t_e < \tilde{t}_d < T_A$.

Case $[0, T_B)$

Suppose that T_B is the first element. Then, for $t \in [T_B, \chi)$ we get (180) and (175), so that (181) - (183) hold. Following the related argument and taking into account that T_A as next element implies no trade for all t yield $\chi = \tilde{t}_d$.

With $\tilde{t}_d$ as next element, we get (153), (170) and (160) for $t \in [\tilde{t}_d, \chi)$, so that (176) - (179) hold. Following Appendix B.1.1, **Case** $[0, t_c), [t_c, t_d), [t_d, T_B), [T_B, \tilde{t}_d)$ yields $\chi = \{T_A, \tilde{T}_B\}$ and, therefore, the timing $0 < T_B < \tilde{t}_d < T_A$.

Case $[0, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B)$ Suppose that $0 < T_B < \tilde{t}_d < \tilde{T}_B < \dots$. Then, for $t \in [\tilde{T}_B, \chi)$, we get (153), (159) and (160), so that (156) and (161) - (163) hold. Because $g_{BB}^s(t) > 0$, $\chi \neq t_c$. Because $g_{BA}^s(t) > 0$, $\chi \neq \{\tilde{t}_d, \tilde{t}_e\}$. Because $g_{AB}^s(t) = 0$, $\chi \neq \{t_e, t_f, \tilde{t}_f\}$. Because $b_A^d(t), b_B^d(t) > 0$, $\chi \neq \{\tilde{T}_A, \tilde{T}_B\}$. Because of Lemma 4, $\chi \neq T_B$. Thus, $\chi = \{\tilde{t}_c, T_A\}$.

Case $[0, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, T_A)$ Suppose that $0 < T_B < \tilde{t}_d < \tilde{T}_B < T_A < \dots$. Then, for $t \in [T_A, \chi)$, we get (164), (159) and (160), so that (165) - (169) hold. The related argument yields t_d and T_B as candidates for χ . In case of t_d , we get (171) and (172), so that (173) and (174) hold between t_d and the next element. According to the related argument, this is T_B^2 . Then, both T_B^1 and T_B^2 are determined by (175), so that the proof of Lemma 4 applies. Consequently, $\chi = T_B$ implying the sequence $0 < T_B < \tilde{t}_d < \tilde{T}_B < T_A < T_B$.

Case $[0, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, \tilde{t}_c)$ Suppose that $0 < T_B < \tilde{t}_d < \tilde{T}_B < \tilde{t}_c < \dots$. Then, for $t \in [\tilde{t}_c, \chi)$, we get (153) - (155), so that (156) - (158) hold. Following the arguments of Appendix B.1.1, **Case** $[0, \tilde{t}_c)$ yields $0 < T_B < \tilde{t}_d < \tilde{T}_B < \tilde{t}_c < T_A < t_c < \dots$, with $t_d, \tilde{T}_A$ and T_B^2 as candidates for the next element. In case of T_B^2 , we get the timing $0 < T_B^1 < \tilde{t}_d < \tilde{T}_B < \tilde{t}_c < T_A < t_c < T_B^2$.

Case $[0, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, \tilde{t}_c), [\tilde{t}_c, T_A), [T_A, t_c), [t_c, t_d)$ Suppose that $0 < T_B < \tilde{t}_d < \tilde{T}_B < \tilde{t}_c < T_A < t_c < t_d < \dots$. Then, for $t \in [t_d, \chi)$, we get (171) and (172), so that (173) and (174) hold. The related argument yields $\chi = T_B^2$. However, both T_B^1 and T_B^2 are determined by (175), so that the proof of Lemma 4 applies. Consequently, this case can be ruled out.

Case $[0, T_B), [T_B, \tilde{t}_d), [\tilde{t}_d, \tilde{T}_B), [\tilde{T}_B, \tilde{t}_c), [\tilde{t}_c, T_A), [T_A, t_c), [t_c, \tilde{T}_A)$ Suppose that $0 < T_B < \tilde{t}_d < \tilde{T}_B < \tilde{t}_c < T_A < t_c < \tilde{T}_A < \dots$. Then, for $t \in [\tilde{T}_A, \chi)$, we get (153), (159) and (160), so that (156) and (161) - (163) hold, with $\dot{g}_{BA}^s > 0$. Following the argument of Appendix B.1.1, **Case** $[0, \tilde{t}_c), [\tilde{t}_c, T_A), [T_A, t_c), [t_c, \tilde{T}_A)$ yields $\chi = T_B^2$. However, the equation system describing

the market equilibrium at $t = T_B^2$ is identical to the one at $t\tilde{T}_B$), so that the proof of Lemma 4 applies. Consequently, this case can be ruled out.

□