A parallel batch greedy algorithm in reduced basis methods

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The classical (weak) greedy algorithm is widely used within model order reduction in order to compute a reduced basis in the offline training phase: An a posteriori error estimator is maximized and the snapshot corresponding to the maximizer is added to the basis. Since these snapshots are determined by a sufficiently detailed discretization, the offline phase is often computationally extremely costly.

We suggest to replace the serial determination of one snapshot after the other by a parallel approach. In order to do so, we introduce a batch size b and add b snapshots to the current basis in every greedy iteration. These snapshots are computed in parallel.

We prove convergence rates for this new batch greedy algorithm and compare them to those of the classical (weak) greedy algorithm in the Hilbert and Banach space case from [1] and [2]. Then, we present numerical results where we apply a (parallel) implementation of the proposed algorithm to the linear elliptic *thermal block* problem. We analyze the convergence rate as well as the offline and online wall-clock times for different batch sizes. We show that the proposed variant can significantly speed-up the offline phase while the size of the reduced problem is only moderately increased. Additionally, the benefit of the parallel batch greedy increases for more complicated problems.

A preprint is available at [3].

References

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