34. Rhein-Ruhr-Workshop

Bestwig, 17.-18. Januar 2025

-Programm, Teilnehmer und Abstracts-

Organisation:

Prof. Dr. G. Plonka-Hoch	Universität Göttingen
Prof. Dr. T. Sauer	Universität Passau
Prof. Dr. M. Skrzipek	FernUniversität in Hagen
Dr. M. Weimar	Universität Würzburg

34. Rhein-Ruhr-Workshop
Bestwig, 17.–18. Januar 2025
PROGRAMM

Freitag, 17. Januar 2025, Vormittag

10.20	*Begrüßung / Organisatorisches*
	Moderation:
10.30	**Frederic Schoppert** (Universität Lübeck) Localized Directional Frames on Spheres
11.00	**Tizian Sommerfeld** (Technische Universität Chemnitz) A Randomized Fast Algorithm for Frame Subsampling
11.30	**Kateryna Pozharska** (Technische Universität Chemnitz) Recent developments in the optimal sampling recovery and discretization problems
12.00	*Gemeinsames Mittagessen und Pause*

Freitag, 17. Januar 2025, Nachmittag

	Moderation:
14.00	**Nick Schneider** (FAU Erlangen-Nürnberg) Adaptive approximation of time-dependent functions with discontinuous anisotropic space-time finite elements
14.30	**Martin Ossadnik** (FernUniversität in Hagen) A posteriori error bounds for pseudo parabolic equations
15.00	**Niklas Reich** (Hochschule Ruhr West) A parallel batch greedy algorithm in reduced basis methods
15.30	*Pause mit Kaffee, Tee, Kuchen*
	Moderation:
16.00	**Max Brockmann** (Universität Köln) Multigrid method for solving elliptic PDEs on metric graphs.
16.30	**Matthias Hofmann** (FernUniversität in Hagen) Computing eigenvalues of the p-Laplacian via graph surgery on combinatorial graphs
17.00	**Florian Boßmann** (Harbin Institute of Technology, China) Improved impedance inversion by iterated graph Laplacian
18.00	*Gemeinsames Abendessen*

Freitag, 17. Januar 2025, ab 19 Uhr:

Präsentation der Poster
Andrei Chernov (Universität Passau) 4D Optical Flow Estimation
Kai Diethelm (Technische Hochschule Würzburg-Schweinfurt) Dirichlet-Type Boundary Value Problems for Fractional Differential Equations and Their Numerical Solution
Safoura Hashemishahraki (Technische Hochschule Würzburg-Schweinfurt) A Constructive Approach for Investigating the Stability of Incommensurate Fractional Differential Systems
Moritz Danzebrink, Janina Tikko (Universität Köln) Finding global minima of non-convex functions via swarm based gradient descent
André Meyer (Universität Köln) Modellierung des Wohlbefindens von Milchkühen mittels Kontrollproblemen
Lena Perlberg, Max Brockmann (Universität zu Köln) Creating migration networks based on archeological data using least-cost-path analysis
Sofie Saier (Universität Lübeck) (Not) Rough Around the Edges – Polynomial Shearlets on the Torus
Serhii Stasyuk (Institute of Mathematics NAS Kiew) Sparse trigonometric approximation of periodic functions from Besov classes with mixed smoothness
Joachim Stöckler (Technische Universität Dortmund) Eine neue Klasse von Shearlets zur Randanpassung

Samstag, 18. Januar 2025

8.00	*Frühstück und anschließendes Räumen der Zimmer, Schlüsselabgabe bis 9:30 Uhr*
	Moderation:
9.00	**Janina Schmidt** (Universität Göttingen) Automated focusing optics adjustment for free-electron lasers
9.30	**Yannick Riebe** (Universität Göttingen) MOCCA: A Fast Algorithm for Parallel MRI Reconstruction Using Model Based Coil Calibration
10.00	**Anahita Riahi** (Uni Göttingen) Reconstruction of Undersampled Fourier Data and Analysis of Modified ACS Sampling
10.30	*Pause mit Kaffee, Tee*
	Moderation:
11.00	**Chaudhary Renu** (Technische Hochschule Würzburg-Schweinfurt) A Diffusive Representation Approach to Fractional Differential Equations
11.30	**Afshin Farhadi** (Technische Hochschule Würzburg-Schweinfurt) Efficient Numerical Methods for Fractional Differential Equations
12.00	*Gemeinsames Mittagessen*

Dauer der Vorträge: 30 Minuten, einschließlich Diskussionszeit.

TEILNEHMERLISTE RRW 2025

Abel, Ulrich	TH Mittelhessen	ulrich.abel@mnd.thm.de
Bossmann, Florian	Harbin Institute of Technology	F.Bossmann@hit.edu.cn
Brockmann, Max	Universität zu Köln	brockmann@math.uni-koeln.de
Chaudhary, Renu	TH Würzburg-Schweinfurt	renu.chaudhary@thws.de
Chernov, Andrei	Universität Passau	andrei.chernov@uni-passau.de
Danzebrink, Moritz	Universität zu Köln	mdanzeb1@uni-koeln.de
Diethelm, Kai	TH Würzburg-Schweinfurt	kai.diethelm@thws.de
Farhadi, Afshin	TH Würzburg-Schweinfurt	afshin.farhadi@thws.de
Filbir, Frank	Helmholz Zentrum Mnchen	frank_filbir@web.de
Hashemishahraki, Safoura	TH Würzburg-Schweinfurt	safoura.hashemishahraki@thws.de
Hofmann, Matthias	FernUniversität in Hagen	matthias.hofmann@fernuni-hagen.de
Kunoth, Angela	Universität zu Köln	akunoth@uni-koeln.de
Meyer, Andr	Universität zu Köln	andre.meyer@uni-koeln.de
Ossadnik, Martin	FernUniversität in Hagen	martin.ossadnik@fernuni-hagen.de
Perlberg, Lena	Universität zu Köln	lperlber@smail.uni-koeln.de
Plonka-Hoch, Gerlind	Universität Göttingen	plonka@math.uni-goettingen.de
Pozharska, Kateryna	Technische Universität Chemnitz	kateryna.pozharska@mathematik.tu-chemnitz.de
Reich, Niklas	Hochschule Ruhr West	niklas.reich@hs-ruhrwest.de
Reif,UIrich	TU Darmstadt	reif@mathematik.tu-darmstadt.de
Riahi, Anahita	Universität Göttingen	a.riahi@math.uni-goettingen.de
Riebe, Yannick	Universität Göttingen	y.riebe@math.uni-goettingen.de
Rieger, Christian	Phillips-Universität Marburg	riegerc@mathematik.uni-marburg.de
Saier, Sofie	Universität zu Lübeck	sofie.saier@student.uni-luebeck.de
Sauer, Tomas	Universität Passau	Tomas.Sauer@uni-passau.de
Schmidt, Janina	Universität Göttingen	j.schmidt@math.uni-goettingen.de
Schneider, Nick	FAU Erlangen-Nürnberg	nick.schneider@fau.de
Schoppert, Frederic	Universität zu Lübeck	f.schoppert@uni-luebeck.de
Skrzipek, Michael	FernUniversität in Hagen	michael.skrzipek@fernuni-hagen.de
Sommerfeld, Tizian	Technische Universität Chemnitz	tizian.sommerfeld@math.tu-chemnitz.de
Stasyuk, Serhii	NAS Ukraine	stasyuk@imath.kiev.ua
Stock, Michael	Universität Passau	stock@forwiss.uni-passau.de
Stöckler, Joachim	Technische Universität Dortmund	joachim.stoeckler@math.tu-dortmund.de
Tarasova, Tatjana	Universität Passau	tatiana.tarasova@uni-passau.de
Tikko, Janina	Universität zu Köln	jtikko2@uni-koeln.de
Weimar, Markus	Universität Würzburg	markus.weimar@uni-wuerzburg.de

Improved impedance inversion by iterated graph Laplacian

Davide Bianchi[1], Florian Boßmann[2], Wenlong Wang[2], Mingming Liu[2]

[1] School of Mathematics, Sun Yat-sen University (Zhuhai), China.
[2] School of Mathematics, Harbin Institute of Technology, China.

Deep learning techniques have shown significant potential in many applications through recent years. The achieved results often outperform traditional techniques. However, the quality of a neural network highly depends on the used training data. Noisy, insufficient, or biased training data leads to suboptimal results. We present a hybrid method that combines deep learning with iterated graph Laplacian and show its application in acoustic impedance inversion which is a routine procedure in seismic explorations. A neural network is used to obtain a first approximation of the underlying acoustic impedance and construct a graph Laplacian matrix from this approximation. Afterwards, we use a Tikhonov-like variational method to solve the impedance inversion problem where the regularizer is based on the constructed graph Laplacian. The obtained solution can be shown to be more accurate and stable with respect to noise than the initial guess obtained by the neural network. This process can be iterated several times, each time constructing a new graph Laplacian matrix from the most recent reconstruction. The method converges after only a few iterations returning a much more accurate reconstruction. We demonstrate the potential of our method on two different datasets and under various levels of noise. We use two different neural networks that have been introduced in previous works. The experiments show that our approach improves the reconstruction quality in the presence of noise.

Modelling Population Dynamics on Networks using Partial Differential Equations solved by Multigrid Methods

Max Brockmann
Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne
brockmann@math.uni-koeln.de

Abstract

I solve partial differential equations on networks with the goal of modelling population dynamics in the Paleolithic and Neolithic, in the context of the interdisciplinary research project HESCOR[1] at the University of Cologne. Modelling human dynamics is restricted by the availability of archaeological data. In order to focus on areas where data is not too sparse for meaningful results, I restrict the domain to a network where vertices are archaeological sites, connected by edges which describe migration corridors. A method for determining these migration corridors is presented in the poster session [BP]. The actual migration dynamics are modelled using a diffusion-advection partial differential equation (PDE).

In order to formulate PDEs on graphs, I explain the network structure with the help of metric graphs. Metric graphs use an edgewise parameterization of the graph, such that differential operators can be defined on them. In order to solve the PDEs on metric graphs, I use a multigrid method [B]. The method is first developed for an elliptic PDE with Neumann-Kirchhoff conditions. The graph is discretized using a finite element (FE) discretization and a hat function basis, as described in [AB]. By combining the discretization with a weak formulation of the elliptic PDE, we can find an approximation to the solution of the PDE in the FE discretization space by solving the system of equations:

$$\begin{pmatrix} \mathbf{H}_{\mathcal{E}\mathcal{E}} & \mathbf{H}_{\mathcal{E}\mathcal{V}} \\ \mathbf{H}_{\mathcal{V}\mathcal{E}} & \mathbf{H}_{\mathcal{V}\mathcal{V}} \end{pmatrix} \mathbf{u} = \mathbf{f},$$

where $\mathbf{u}$ is the coefficient vector of the solution of the PDE written in its basis.

This system requires an efficient solver, because every edge of the metric graph is discretized using $n_e \in \mathbb{N}$ discretization points. Consequently, for sufficiently complex graphs, the discretization is applied to a large set of edges, resulting in a large system of equations. I use a multigrid method and develop suitable intergrid operators for the solution of this system of equations. Using a semi-discretization FE, this method can be extended to parabolic PDEs ([W]).

References

[AB] M. Arioli, M. Benzi, *A finite element method for quantum graphs*, IMA Journal of Numerical Analysis, Volume 38, Issue 3, Pages 1119-1163, 2017.

[B] M. Brockmann, *Solving Elliptic PDEs on Metric Graphs: Finite Element Discretization, Multigrid Method and PCG Solver*, Master Thesis, Department of Mathematics and Computer Science, Division of Mathematics, 2023.

[BP] M. Brockmann, L. Perlberg, *Creating migration networks based on archaeological data using least-cost-path analysis*, Poster, Rhein-Ruhr-Workshop 2025.

[W] A. Weller, *Numerical Methods for Parabolic Partial Differential Equations on Metric Graphs*, Ph.D. Thesis, Faculty of Mathematics and Natural Sciences, University of Cologne, 2024

Funded by:
HESCOR Human & Earth System Coupled Research
Ministry of Culture and Science of the State of North Rhine-Westphalia

[1] HESCOR: Human & Earth System Coupled Research, https://hescor.uni-koeln.de

Creating Migration Networks based on Archaeological Data using Least-Cost-Path Analysis

Max Brockmann
brockmann@math.uni-koeln.de

Lena Perlberg
lperlber@smail.uni-koeln.de

Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne

Abstract

We investigate how to identify possible migration routes of humans in the past, represented as networks. Human's presence spanned large parts of Europe in the time periods from the Upper Paleolithic to the Neolithic. We model major migratory events using migration corridors between a map of human presence sites derived from archaeological data. The question of creating migration networks arises is the context of the interdisciplinary research project HESCOR[1] at the University of Cologne.

We use data provided by archaeologists([KH],[M]). These datasets form the basis for human presence. Each archaeological site serves as a guaranteed marker of human presence and thus represents a node in the network. Additionally to the location of the sites, the data includes information on materials found. This provides valuable insights into cultural aspects, which are used in the creation of connections between the nodes.

We use the distance between nodes of the network as the primary factor of possible connectivity. In essence, we propose a model of connectivity in which the probability of connection increases as the distance between nodes decreases. Cultural information is incorporated by utilizing data on shared material presence from archaeological findings. If specific materials are only produced in one area, but are found throughout Europe, these must have been transported by humans. This indicates a higher probability of connection between two locations and is therefore included in the stochastic decision rule.

We base our distance calculation on the concept of effective distance. We argue that an effective distance approach best mimics migration corridors, as humans would have had to base their migration decisions on a relation of distance to effort required. To calculate the effective distance, we determine a least-cost path using Dijkstra's algorithm applied to an elevation raster using topographic data ([ETOPO]). We allow king's movement (as in chess) between raster cells and calculate the movement costs with a modified Tobler's hiking function from [KS], which maps slope to walking speed. This least-cost path is calculated for each pair of nodes. Subsequently, probability-based decision-making process is applied. After implementing all concepts, we can generate graphs that describe a network of possible human migration routes in the past.

We present current results of migration networks for different time periods and analyse the effects of including material data provided by archaeologists.

References

[ETOPO] NOAA National Centers for Environmental Information. 2022: ETOPO 2022 15 Arc-Second Global Relief Model.

[KH] T. Kerig, J. Hilpert et al., *Interlinking research: the Big Exchange project*, Antiquity 97, no. 394, 2023.

[KS] Y. Kondo, Y. Seino, *GPS-aided Walking Experiments and Data-driven Travel Cost Modeling on the Historical Road of Nakasendō-Kisoji (Central Highland Japan)*, Computer Applications and Quantitative Methods in Archaeology, Proceedings of the 37th International Conference, pp. 158-165, 2010.

[M] A. Meier, *The Central European Magdalenian*, Springer Dordrecht, 2015.

[1] HESCOR: Human & Earth System Coupled Research, https://hescor.uni-koeln.de

HESCOR Human & Earth System Coupled Research

Funded by:
Ministry of Culture and Science of the State of North Rhine-Westphalia

A DIFFUSIVE REPRESENTATION APPROACH TO FRACTIONAL DIFFERENTIAL EQUATIONS

RENU CHAUDHARY
TECHNISCHE HOCHSCHULE WÜRZBURG-SCHWEINFURT

Abstract. Fractional calculus is a powerful mathematical framework for modeling complex systems across fields such as physics, engineering, and finance. However, the numerical computation of fractional integrals and solving fractional differential equations presents significant challenges due to the non-local nature of fractional operators, leading to high computational and memory costs in traditional methods. In this study, we explore innovative variations in diffusive representations specifically designed for fractional integrals, aiming to reduce computational complexity and memory usage. By integrating schemes like the Gauss-Laguerre quadrature scheme and kernel compression, we develop enhanced numerical methods for solving fractional differential equations.

The talk is based on joint work with Kai Diethelm, Afshin Farhadi and Fred Fuchs from Technische Hochschule Würzburg-Schweinfurt.

References

[1] Diethelm, K. (2022). A new diffusive representation for fractional derivatives, Part II: Convergence analysis of the numerical scheme. Mathematics, 10(8), 1245.

[2] Diethelm, K. (2023). Diffusive representations for the numerical evaluation of fractional integrals. In 2023 International Conference on Fractional Differentiation and Its Applications (ICFDA) (pp. 1-6). IEEE.

[3] Chaudhary, R., and Diethelm, K. (2024). Novel variants of diffusive representation of fractional integrals: Construction and numerical computation. IFAC-PapersOnLine, 58(12), 412-417.

[4] Chaudhary, R., and Diethelm, K. (2024). Revisiting diffusive representations for enhanced numerical approximation of fractional integrals. IFAC-PapersOnLine, 58(12), 418-423.

4D Optical Flow Estimation

Andrei Chernov

University of Passau

4D Optical Flow aims to estimate the motion between two 3D volumetric datasets captured over time. This process involves calculating the displacement of each voxel in a 3D space from one time frame to the next. Although traditional 2D optical flow estimates motion between two 2D images, 4D optical flow extends this concept to include three spatial dimensions (X, Y, Z) as well as the temporal dimension (T).

Finding global minima of non-convex functions via swarm based gradient descent

Moritz Danzebrink
mdanzeb1@uni-koeln.de

Janina Tikko
jtikko2@uni-koeln.de

Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne

Abstract

We present a Swarm-Based Gradient Descent method (SBGD) introduced in [JTZ]. The underlying problem is to find global minima and can be formulated as

$$\operatorname*{argmin}_{\mathbf{x}\in\Omega\subset\mathbb{R}^d} f(\mathbf{x})$$

for a function $f : \Omega \subset \mathbb{R}^d \to \mathbb{R}$. The challenge is to overcome basins of local minima in order to find the true global minima. Other methods like classical Gradient Descent depend heavily on starting positions, hence, they struggle to escape local minima. Our presented approach combines Gradient Descent with swarm-behavior in order to overcome the difficulties of local minima.

A swarm contains a number of agents which are each defined by a position $\mathbf{x}$ and a mass m. The agents explore the ambient space by moving along the direction of the gradient. As stepsize-protocol the backtracking-line-search method is used, as proposed by Tadmor et al [JTZ]. Key aspect in this method is communication between agents through transferring parts of their mass. Each agent receives a different stepsize $h = h(\mathbf{x}, m)$, depending on its individual mass. This leads to lighter agents exploring regions which are further away. On the contrary heavier agents receive smaller stepsizes approaching potential minima. Naturally the swarm is divided in lighter explorers which improve the swarms overall position and a global leader. The global leader might change through the process due to an improved minimizer.

During our research we demonstrated the operating principle [T]. We give a more visual understanding of how the new method works and how different parameters impact the algorithm. In particular we analyzed the influence of the relative mass on the backtracking protocol [D]. Moreover, we studied the quality of the error with respect to the number of agents.

References

[D] M. Danzebrink, *Optimization approach for Swarm-Based Gradient Descent in multiple arguments*, Bachelor Thesis, Department of Mathematics and Computer Science, Division of Mathematics, University of Cologne, November 2024.

[JTZ] L. Jingcheng, E. Tadmor, A. Zenginoğlu, *Swarm-based gradient descent method for non-convex optimization*, Communications of the American Mathematical Society, Volume 4, Pages 787-822, 2024, doi:10.1090/cams/42.

[T] J. Tikko, *Introduction: Swarm-based gradient descent for non convex optimization*, 2024, https://arxiv.org/abs/2404.00005.

Dirichlet-Type Boundary Value Problems for Fractional Differential Equations and Their Numerical Solution

Kai Diethelm
Faculty of Applied Natural Sciences and Humanities (FANG)
Technical University of Applied Sciences Würzburg-Schweinfurt
kai.diethelm@thws.de

We investigate Dirichlet-type boundary value problems associated to nonlinear fractional differential equations of order $\alpha \in (1, 2)$ that use Caputo derivatives. In particular, we discuss the existence and uniqueness of solutions and we propose a numerical solution algorithm based on employing shooting methods. Specifically, we demonstrate that the so-called proportional secting technique for selecting the required initial values leads to numerical schemes that converge to high accuracy in a very small number of shooting iterations, and we provide an explanation of the analytical background for this favourable numerical behaviour. Moreover, we point out some open questions in connection with the location of certain zeros of Mittag-Leffler functions that play an important role in this context.

EFFICIENT NUMERICAL METHODS FOR FRACTIONAL DIFFERENTIAL EQUATIONS

AFSHIN FARHADI
TECHNISCHE HOCHSCHULE WÜRZBURG-SCHWEINFURT

Abstract. Fractional differential equations (FDEs) play a pivotal role in modeling complex systems characterized by memory and hereditary properties. However, the numerical evaluation of fractional operators remains computationally challenging due to their non-local nature. In this study, we introduce an advanced and efficient numerical approach based on diffusive representations and the double exponential (DE) formula for numerical integration. The DE formula is adapted to efficiently compute fractional integrals by addressing the rapid decay of the integrand in diffusive representations. This method achieves high accuracy with reduced computational complexity, offering a significant improvement over conventional techniques. The proposed method provides a powerful tool for solving FDEs with enhanced accuracy and efficiency, making it suitable for a wide range of scientific and engineering applications.

The talk is based on joint work with Kai Diethelm, Renu Chaudhary and Fred Fuchs from Technische Hochschule Würzburg-Schweinfurt.

References

[1] Diethelm, K. (2022). A new diffusive representation for fractional derivatives, Part II: Convergence analysis of the numerical scheme. Mathematics, 10(8), 1245.
[2] Diethelm, K. (2023). Diffusive representations for the numerical evaluation of fractional integrals. In 2023 International Conference on Fractional Differentiation and Its Applications (ICFDA) (pp. 1-6). IEEE.
[3] Chaudhary, R., and Diethelm, K. (2024). Novel variants of diffusive representation of fractional integrals: Construction and numerical computation. IFAC-PapersOnLine, 58(12), 412-417.
[4] Chaudhary, R., and Diethelm, K. (2024). Revisiting diffusive representations for enhanced numerical approximation of fractional integrals. IFAC-PapersOnLine, 58(12), 418-423.

Computing eigenvalues of the p-Laplacian via graph surgery on combinatorial graphs

Matthias Hofmann

Abstract

We develop a perturbation theory of the discrete signed p-Laplacian under perturbation by a cut parameter, developing a Hellman-Feynman type theorem for the derivative in the nonlinear setting $p \neq 2$. Using an approach based on graph surgery, we develop a model based on such a perturbation of the p-Laplacian on a cut graph to study the eigenvalue problem in the original setting. We show that the eigenvalues of the signed p-Laplacian can be characterized via extremal points of the constructed perturbed system. In this context, we elaborate on how graph surgery can be used in order to compute eigenvalues of the (signed) p-Laplacian by looking at some examples. Based on a joint work with Gregory Berkolaiko.

Modellierung des Wohlbefindens von Milchkühen mittels Kontrollproblemen

André Meyer
Department of Mathematics and Computer Science, Division of Mathematics
University of Cologne
andre.meyer@uni-koeln.de

Abstract

Ich modelliere die Temperatur in einem landwirtschaftlichen Gebäude, in dem Milchvieh gehalten wird. Da Milchkühe fast das ganze Jahr über in diesem Gebäude zur Milchabgabe leben, sind sie dort klimatischen Bedingungen ausgesetzt. Ziel ist es, den Hitzestress (und Kältestress) in Milchviehbetrieben zu reduzieren, indem die Temperatur kontrolliert und angepasst wird. Dazu wurde ein Thermometer im Gebäude als auch außerhalb installiert, um erste Daten zu erheben. Weiter gibt es eine junge Arbeit [1] die zeigt, dass sich sowohl Hitzestress als auch Kältestress auf die Milchleistung von Milchkühen auswirkt. Diese Arbeit berichtet von einer optimalen Temperatur von 19°C. Die Idee meiner Arbeit ist es, mittels der Wärmeleitungsgleichung ein Kontrollproblem zu entwickeln und zu lösen.

References

[1] E. Choi, V. Carneiro de Souza, J.A. Dillon, E. Kebreab, N.D. Mueller, *Comparative analysis of thermal indices for modeling cold and heat stress in US dairy systems*, Journal of Dairy Science, Volume 107, Issue 8, August 2024, Pages 5817-5832.

A posteriori error bounds for pseudo parabolic problems

Martin Ossadnik* Torsten Linß *

We consider a third-order pseudo parabolic equation of finding $u : [0, T] \mapsto H_0^1(\Omega)$, $\Omega \subset \mathbb{R}^d$, such that

$$\mathcal{L}\partial_t u(t) + \mathcal{M}u(t) = F(t) \quad \text{in } (0, T] \tag{1}$$

with two second order, elliptic operators $\mathcal{L}, \mathcal{M} : H_0^1(\Omega) \mapsto H^{-1}(\Omega)$ and a source function $F : [0, T] \mapsto H^{-1}(\Omega)$. Furthermore an initial condition

$$u(0) = u_0, \quad u_0 \in H_0^1(\Omega), \tag{2}$$

is given. We will assume, that the operator $\mathcal{M}$ is bounded and that $\mathcal{L}$ is both bounded and coercive.

A computable $L^\infty\left(0, T; H_0^1(\Omega)\right)$ a posteriori error bound for a full discretisation, using the backward differential formula of order two (BDF-2 method) in time and $\mathbb{P}_2$-elements in space, is derived. To do so, we leverage the C_0 semigroup, generated by the operator $\mathcal{L}^{-1}\mathcal{M}$, and adapt elliptic reconstructions introduced by C. Makridakis and R. N. Nochetto to pseudo parabolic problems.

Given some numerical results we analyze the estimate's order, efficiency and components. We show that we can apply our a posteriori error bound to other time discretisations like the backward Euler and Crank Nicolson method.

References

[1] Ch. Makridakis and R. H. Nochetto. Elliptic reconstruction and a posteriori error estimates for parabolic problems. *SIAM J. Numer. Anal.*, 41(4):1585–1594, 2003.

*Fakultät für Mathematik und Informatik, FernUniversität in Hagen, Universitätsstraße 11, 58095 Hagen, Germany, [martin.ossadnik, torsten.linss]@fernuni-hagen.de

Recent developments in the optimal sampling recovery and discretization problems

Kateryna Pozharska

Institute of Mathematics of the National Academy of Sciences of Ukraine,
Faculty of Mathematics of Chemnitz University of Technology, Germany

In the talk, we discuss new developments in the power of different types of information (function values vs. linear measurements, deterministic vs. random) as well as different classes of algorithms (linear vs. nonlinear).

The main emphasis will be made on the uniform sampling recovery of bounded complex-valued functions. Besides, we will show how to lift L_2-error bounds to error bounds in general semi-normed spaces using the spectral function. In what follows, we discuss related problems of the discretization of continuous norms, in particular of the uniform norm, and recent findings on the tight Marcinkiewicz–Zygmund inequalities.

Based on the joint work with Felix Bartel (UNSW Sydney), David Krieg (University of Passau), Martin Schäfer (TU Chemnitz), Mario Ullrich (JKU Linz) and Tino Ullrich (TU Chemnitz).

A parallel batch greedy algorithm in reduced basis methods

Niklas Reich, Jürgen Vorloeper
Hochschule Ruhr West, Institut Naturwissenschaften
niklas.reich@hs-ruhrwest.de

Karsten Urban
Universität Ulm, Institut für Numerische Mathematik

The classical (weak) greedy algorithm is widely used within model order reduction in order to compute a reduced basis in the offline training phase: An a posteriori error estimator is maximized and the snapshot corresponding to the maximizer is added to the basis. Since these snapshots are determined by a sufficiently detailed discretization, the offline phase is often computationally extremely costly.

We suggest to replace the serial determination of one snapshot after the other by a parallel approach. In order to do so, we introduce a batch size b and add b snapshots to the current basis in every greedy iteration. These snapshots are computed in parallel.

We prove convergence rates for this new batch greedy algorithm and compare them to those of the classical (weak) greedy algorithm in the Hilbert and Banach space case from [1] and [2]. Then, we present numerical results where we apply a (parallel) implementation of the proposed algorithm to the linear elliptic *thermal block* problem. We analyze the convergence rate as well as the offline and online wall-clock times for different batch sizes. We show that the proposed variant can significantly speed-up the offline phase while the size of the reduced problem is only moderately increased. Additionally, the benefit of the parallel batch greedy increases for more complicated problems.

A preprint is available at [3].

References

[1] P. Binev, A. Cohen, W. Dahmen, R. DeVore, G. Petrova, and P. Wojtaszczyk. “Convergence rates for greedy algorithms in reduced basis methods”. *SIAM J. Math. Anal.* 43.3 (2011), pp. 1457–1472.

[2] R. DeVore, G. Petrova, and P.Wojtaszczyk. “Greedy algorithms for reduced bases in Banach spaces”. *Constr. Approx.* 37.3 (2013), pp. 455–466.

[3] N. Reich, K. Urban, J. Vorloeper. “A parallel batch greedy algorithm in reduced basis methods: Convergence rates and numerical results”. 2024, *arXiv:* 2407.11631 [math.NA].

Reconstruction of Undersampled Fourier Data and Analysis of Modified ACS Sampling:

Anahita Riahi, Georg-August-Universität Göttingen

In applications such as Magnetic Resonance Imaging (MRI), data are acquired in the Fourier domain and to accelerate imaging, a subsample of the discrete Fourier data is often taken. In order to recover the image, we need to reconstruct the missing Fourier data and then apply the discrete inverse Fourier transform. However, reconstructing missing data in the Fourier Space poses significant challenges. Unlike the image domain, the Fourier domain is not localised, which means the data are not as easily reconstructed. In this talk, we will see why linear interpolation in the Fourier domain is futile and discuss alternative reconstruction methods, namely "Generalized Autocalibrating Partially Parallel Acquisitions" (GRAPPA), a widely adopted approach in MRI reconstruction. GRAPPA leverages the "Auto-Calibration Signal" (ACS) region, which is an area in the low-pass part of the Fourier data that is fully sampled. In GRAPPA and many other MRI methods, data outside of the ACS region are often subsampled with a constant subsampling rate of R. We will examine the critical role of subsampling strategies outside the ACS region and how these influence reconstruction quality and we will consider alternative subsampling methods to utilise GRAPPA to its full capacity.

MOCCA: A Fast Algorithm for Parallel MRI Reconstruction Using Model Based Coil Calibration

Yannick Riebe
University of Göttingen
y.riebe@math.uni-goettingen.de

Parallel Magnetic Resonance Imaging (MRI) based on simultaneous measurements from multiple receiver coils has been introduced to overcome the relatively slow data acquisition time and at the same time, to achieve improved high-resolution images. To achieve the wanted acceleration of the acquisition time, the goal is to reconstruct the high-resolution proton density (the image) from a subsampled amount of data, thereby exploiting the information from the parallel receiver channels. In the model we use, it is assumed that the given data for each coil is a subsample of the Fourier transform of the product of the proton density and the respective coil sensitivity function. Unfortunately, in general, the coil sensitivity functions are also not known beforehand and have to be estimated from the measured data.

In this talk, I will introduce a new MOdel-based Coil CAlibration (MOCCA) algorithm to reconstruct the coil sensitivities and the proton density from the given (incomplete) measurements. Our new method employs the assumption that the coil sensitivities are smooth functions which can be represented as bivariate trigonometric polynomials of small degree while the proton density is only assumed to be a compactly supported distribution. I will derive fast algorithms for the case of complete and incomplete data that perfectly reconstruct the proton density as well as all sensitivities for the case that they satisfy the considered model exactly. Moreover, I will show that the model fits real MRI data sufficiently well, such that it can be employed for parallel MRI reconstruction in practice.

This talk is based on a joint work with Gerlind Plonka-Hoch.

A CONSTRUCTIVE APPROACH FOR INVESTIGATING THE STABILITY OF INCOMMENSURATE FRACTIONAL DIFFERENTIAL SYSTEMS

SAFOURA HASHEMISHAHRAKI
TECHNISCHE HOCHSCHULE WÜRZBURG-SCHWEINFURT

ABSTRACT. In this work, we study the asymptotic behaviour of solutions to generalized incommensurate fractional systems. We consider fractional systems with rational orders and introduce a constructive approach for analyzing the stability of incommensurate fractional order systems with rational orders in arbitrary dimensions. It introduces a criterion to determine the asymptotic stability of the system.

This presentation is based on joint work with Kai Diethelm from Technische Hochschule Würzburg-Schweinfurt, Ha Duc Thai, and Hong The Tuan from Academy of Science and Technology Vietnam.

(Not) Rough Around the Edges – Polynomial Shearlets on the Torus

Sofie Saier (Universität zu Lübeck)

Abstract

Edge detection represents an important task in signal processing. But while jump discontinuities in onedimensional data are entirely characterized by their location, the formulation of corresponding higherdimensional problems might include questions about local geometric properties of an edge, such as orientation or curvature.

We present an approach to characterize certain types of smooth edges on the twodimensional torus via shearlet transformation which was inspired by work on the real plane. In our periodic setting, the utilized shearlet system consists of trigonometric polynomials. Their corresponding shearlet coefficients allow conclusions concerning location and orientation of edges by their decay behavior.

The discussed construction and results can be found in [1].

References

[1] K. Schober, J. Prestin, and S. Stasyuk. "Edge detection with trigonometric polynomial shearlets". In: *Advances in Computational Mathematics* 47 (Feb. 2021).

The investigation of these results is part of my master's thesis. The presentation is therefore given within the context of my progress at the time of the conference.

Automated Focusing Optics Adjustment for Free-electron Lasers

Janina Schmidt

Georg-August Universität Göttingen
Project with: Gerlind Plonka-Hoch, Klaus Mann, Bernd Schäfer

FLASH is a free-electron laser capable of emitting femto second short pulses of light in the x-ray spectrum. Before the beam is used in experiments, it should be focused. This is done by a Kirkpatrick-Baez mirror system which consists of two mirrors that can be bent, rotated, and translated. At the moment this mirror system has to be tuned by hand before each experiment, which is very time-consuming. The goal is to find a method to choose the 12 parameters of the KB optics automatically, depending on the varying properties of the incoming beam and on the experiment's requirements. We present our model for simulating the propagation through the mirror system as well as methods to solve the resulting optimization problem.

Adaptive approximation of time-dependent functions with discontinuous anisotropic space-time finite elements

Nick Schneider (FAU Erlangen-Nürnberg)

January 17, 2025

We study the approximation of real-valued functions $f : [0, T] \times \Omega \to \mathbb{R}$ with different smoothness in time and space for $T \in (0, \infty)$ and a d-dimensional, simplicial Lipschitz domain Ω.

Starting with a tensor product partition consisting of prisms, we apply a certain sequence of iterations of spacial and temporal bisections, that we call anisotropic bisection method, to refine the initial partition. On the refined partition, we approximate the given function f with discontinuous anisotropic finite elements.

This is used in order to give an almost characterization of the corresponding approximation classes in terms of anisotropic Besov spaces.

The talk is based on joint work with Pedro Morin (Universidad Nacional del Litoral, Santa Fe) and Cornelia Schneider (Friedrich-Alexander-Universität Erlangen-Nürnberg, Erlangen).

Localized Directional Frames on Spheres

Frederic Schoppert
f.schoppert@uni-luebeck.de

Institute of Mathematics, University of Lübeck

Abstract

In the existing literature, the study of localized polynomial frames on higher dimensional spheres is usually restricted to the case of isotropic wavelets. In this talk, we construct a wide class of frames for $L^2(\mathbb{S}^d)$ which includes the well known zonal systems, but also allows for anisotropic designs. In particular, we demonstrate how one can obtain discretized polynomial wavelets that are well localized in space and highly directional.

A Randomized Fast Algorithm for Frame Subsampling

Martin Schäfer* Tizian Sommerfeld*

Chemnitz University of Technology, Department of Mathematics

Abstract

Lee and Sun [LS17] constructed a new randomized algorithm for fast frame subsampling reducing the problem to solving a semi-definite program. We continue their work and show that an almost equal algorithm with deterministic properties exists, such that precise results, which do not depend on random terms, are possible. Moreover, we generalize to non-tight frames as input and elaborate ways to speed up the program.

References

[LS17] Y. T. Lee and H. Sun. "An SDP-Based Algorithm for Linear-Sized Spectral Sparsification". In: (2017). URL: doi.org/10.48550/arXiv.1702.08415.

*E-mail adresses: martin.schaefer@mathematik.tu-chemnitz.de
tizian.sommerfeld@mathematik.tu-chemnitz.de

SPARSE TRIGONOMETRIC APPROXIMATION OF PERIODIC FUNCTIONS FROM BESOV CLASSES WITH MIXED SMOOTHNESS

SERHII STASYUK

Our main interest in this talk is to study sparse trigonometric approximation for periodic multivariate functions from Besov classes with mixed smoothness. We use techniques connected with hyperbolic cross approximation. Motivated by recent results on the connection to nonlinear sampling recovery, we obtain order bounds and tractable bounds for the best m-term trigonometric approximation of the mentioned classes.

This is based joint work with Moritz Moeller and Tino Ullrich.

References

[1] M. Moeller, S. Stasyuk, and T. Ullrich, *High-dimensional sparse trigonometric approximation in the uniform norm and consequences for sampling recovery.* arXiv:2407.15965, [math.NA] 22 Jul 2024.

Technical University Chemnitz (Germany),
Institute of Mathematics of NASU (Ukraine),
stas.serg.a@gmail.com stasyuk@imath.kiev.ua

Eine neue Klasse von Shearlets zur Randanpassung

Joachim Stöckler, TU Dortmund

Ergebnisse einer gemeinsamen Arbeit mit M. Weimann werden vorgestellt. Wir behandeln eine neue Klasse von Shearlets, die für die Bildverarbeitung geeignet sind. Dazu werden aus einer gewöhnlichen diskreten Shearlet-Familie in 2D diejenigen Anteile entfernt, die zu horizontalen und vertikalen Elementen, also zu den Elementen mit verschwindendem Scherungsparameter gehören. An ihre Stelle werden tight Framelets aus B-Splines eingefügt, deren Träger genau auf das Bild-Rechteck eingestellt wird. So werden die Rand-Artefakte der üblichen Shearlet-Transformation im FFST-Algorithmus deutlich reduziert.

[1] S. Häuser, G. Steidl, Fast Finite Shearlet Transform: a tutorial. arXiv:1202.1773v2 [math.NA] 23 Jul 2014.