Dirichlet-Type Boundary Value Problems for Fractional Differential Equations and Their Numerical Solution

Kai Diethelm

Faculty of Applied Natural Sciences and Humanities (FANG)
Technical University of Applied Sciences Würzburg-Schweinfurt
kai.diethelm@thws.de

We investigate Dirichlet-type boundary value problems associated to nonlinear fractional differential equations of order $\alpha \in (1, 2)$ that use Caputo derivatives. In particular, we discuss the existence and uniqueness of solutions and we propose a numerical solution algorithm based on employing shooting methods. Specifically, we demonstrate that the so-called proportional secting technique for selecting the required initial values leads to numerical schemes that converge to high accuracy in a very small number of shooting iterations, and we provide an explanation of the analytical background for this favourable numerical behaviour. Moreover, we point out some open questions in connection with the location of certain zeros of Mittag-Leffler functions that play an important role in this context.